Concept: Fair Division
Study Abstract: Fair Division

The subject of fair division deals with situations where a number of people have to divide one or more items among themselves in a way that guarantees everyone gets a "fair share." There are many variations depending on how many people are involved, whether an individual item can be cut up without destroying its value, and even what is meant by a "fair share."

One of the things that makes such problems challenging—and a fact that you should keep in mind—is that each person may place a different value on an item or part of an item. Also, by a "fair share" we will mean that for two people each is entitled to at least half of the total value, for three people each is entitled to at least a third of the total value, etc. In the examples that follow, you will be one of the participants and the computer will play the part of the other people. You should think of this as a game where you want to get as much for yourself as possible—but you definitely want to make sure that you get your fair share of a half, a third, etc. Remember, only you will know how much you value an item, so only you can decide what is a fair share.

Concept  Land Division
Lesson: Lake Front Property Land Division

You get to share a valuable piece of land, located between a lake and a road, with some other people. As you play this game, it might be helpful to imagine that possibly the west side of the property has the best beach for swimming, the east side has the best place for a boat dock, and the best site for building a cabin is in the middle. So determining the value of a piece of land is not merely calculating an area or measuring the length of the waterfront.

For simplicity, the land will be divided by cuts made to the road. Each person will be given a color—your color is green. To make your cut you will position your cursor over the land and then click and drag to define and move your cut line.

Your computer will play the part of the other people who make their cuts with different colors. To help understand how to handle this problem, begin by playing the two person game several times before moving on to the three person game.

Abstract  Two Person Game
The Two Person Land Division Game

(Please refresh page if game does not automatically reload.)

With two people, you want to make sure that you get at least a half of the value. On the map above, make your cut so that you think both the left and right sides are worth a half. Your computer will then mark in black the position it believes divides the property into two equal pieces, worth half each. Since you and your computer probably have different opinions, your marks will be in different places.
The Two Person Land Division Game

(Please refresh page if game does not automatically reload.)

Looking at the map, can you see a way to give both yourself and the computer a satisfactory piece of Land? Click on the land to see if you are correct.
The Two Person Land Division Game

(Please refresh page if game does not automatically reload.)

The piece of land shaded green is a piece that you picked when you made your cut and so is worth a half to you. Similarly, the gray shaded piece was cut to be worth a half to your computer. So you can each get a satisfactory piece and still have some land in the middle left over. You could now each take part of this land and you would both have more than a half.

What makes this allocation of the land satisfactory? Both people get a piece that they themselves marked and choose to be worth a half.

Click "Play Again" to play the two person game again to make sure you understand the principles involved here.

Click "Play The Three Person Game" to go on to the three person game.
The Three Person Land Division Game - Introduction

Each person makes two cuts, dividing the land into three acceptable pieces.

For example, Green, Black and Red might divide the land as follows.

Then the acceptable areas for each of the players are as shown below:

The areas highlighted in blue allow for the easiest distribution. Any extra land will be distributed through the same division process.
The Three Person Land Division Game

With three people, you want to make sure that you get at least one third of the value. Which color should receive section A?

- RED
- GREEN
- BLACK

Three Person Game Intro  Exercise 1
**The Three Person Land Division Game**

**CORRECT!** The leftmost cut defines a piece that GREEN believes to be worth a third, so that leftmost piece can be assigned to GREEN. Here the leftmost piece has been assigned and any other cuts made by that person have been removed since that person has a satisfactory piece.

Now, the rightmost cut defines a piece that another person believes to be worth a third.

Which color should receive the rightmost section (F)?

- RED
- BLACK
The Three Person Land Division Game

CORRECT! The rightmost cut defines a piece that RED believes to be worth a third, so that rightmost piece can be assigned to RED. Here the rightmost piece has been assigned and any other cuts made by that person have been removed since that person has a satisfactory piece.

Now looking in the middle, the final two remaining cuts were made by the same person (BLACK), so the middle can be assigned to that person.

Note that each person has a piece that must be satisfactory since that is the way they marked it in the beginning. Also, since there is land left over (sections B and D), we could give each person some of this leftover land so that everyone would actually get more than a third of the value.
The Land Division Game Summary

In the Land Division Game, the essential thing to notice is that every player is satisfied because each person got a piece of land he or she marked. The idea can be extended to any number of players. In general, as long as there is disagreement there will be extra land left over so that the players can get even more than what they marked as fair shares. In most fair division problems, disagreement is a good thing.

Two Person Game  Exercise 1
Exercise 1

Which color should receive section G?

- RED
- GREEN
- BLACK
**The Three Person Land Division Exercise**

CORRECT! The rightmost cut defines a piece that someone believes to be worth a third, so that rightmost piece can be assigned to BLACK. Here the rightmost piece has been assigned and any other cuts made by that person have been removed since that person has a satisfactory piece.

Now, the leftmost cut defines a piece that another person believes to be worth a third.

Which color should receive the leftmost section (A)?

- [ ] RED
- [ ] GREEN
The Three Person Land Division Exercise

CORRECT! The leftmost cut defines a piece that someone believes to be worth a third, so that leftmost piece can be assigned to RED.

Now looking in the middle, the final two remaining cuts were made by the same person (GREEN), so the middle can be assigned to that person.
Exercise 2

Which sections will be received by RED?

- B
- F
- A and B
- F and G
- C, D and E
**CORRECT!** The leftmost cut defines a piece that someone believes to be worth a third, so that leftmost piece can be assigned to GREEN. The rightmost cut defines a piece that someone believes to be worth a third, so that rightmost piece can be assigned to BLACK.

Now looking in the middle, the final two remaining cuts were made by the same person (RED), so the middle can be assigned to that person.