On the integration of one term of the surname function

Your Name

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title

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In school we usually use the fundamental theorem of calculus to evaluate integrals.

Theorem

Fundamental Theorem of Calculus *Suppose that F is a differentiable function on an interval (a, b), is continuous on $[a, b]$, and $F'(x) = f(x)$ for all $x \in (a, b)$. Then*

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

Evaluating Integrals

In view of the fundamental theorem, we can evaluate $\int_a^b f(x)\,dx$ as follows.

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Obviously, the difficulty is guessing the function F. One trick to help us guess is *u substitution*.

U Substitution

Example

Given coefficents a_0 and a_1, evaluate $\int_0^1 \sin\left(a_1 \pi x + a_0\right)\, dx$.

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Define $u(x) = a_1 \pi x + a_0$. Then $du = a_1 \pi\, dx$, and

$$\begin{aligned}
\int_0^1 \sin(a_1 \pi x + a_0)\, dx &= \frac{1}{a_1 \pi} \int_{u(0)}^{u(1)} \sin u\, du \\
&= \frac{1}{a_1 \pi} (-\cos u) \Big|_{u(0)}^{u(1)} \\
&= \frac{\cos(u(0)) - \cos(u(1))}{a_1 \pi} \\
&= \frac{\cos(a_0) - \cos(a_1 \pi + a_0)}{a_1 \pi}.
\end{aligned}$$