Fast stable computation of the eigenvalues of unitary-plus-rank-one matrices, including companion matrices

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Washington State University

AMS Sectional Meeting, April 22, 2017

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- unitary-plus-rank-one?

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- Who's "we"?

Our International Research Group

This is joint work with

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as well as

- **Leonardo Robol** (Pisa $\rightarrow$ KU Leuven $\rightarrow$ Pisa)

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- ... but we will focus on the companion *matrix* problem today.

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  - several methods proposed

Some of the Competitors

- Chandrasekaran, Gu, Xia, Zhu (2007)
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- Our method is faster, and we can prove backward stability.

Storage Scheme, Part I

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Store Hessenberg matrix in QR decomposed form

$$A = QR$$

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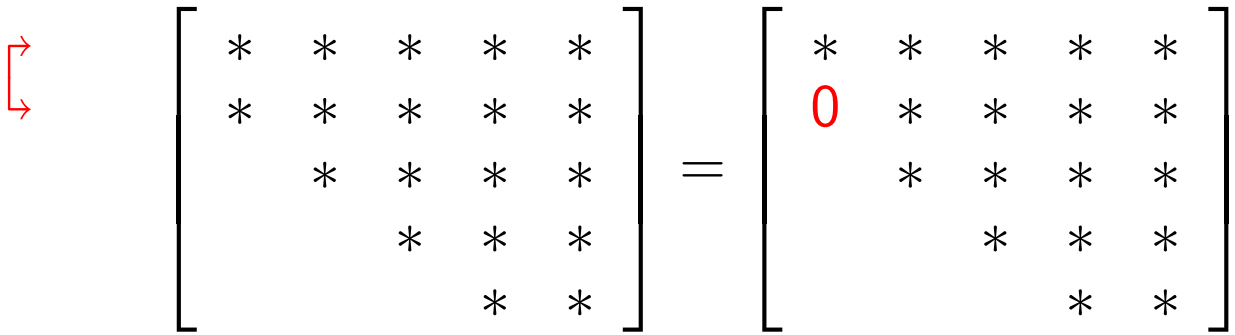
Storage Scheme, Part I



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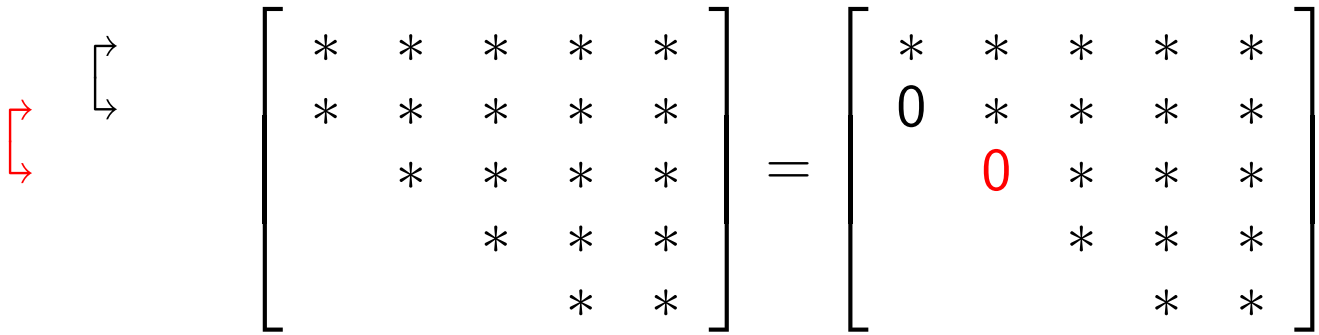

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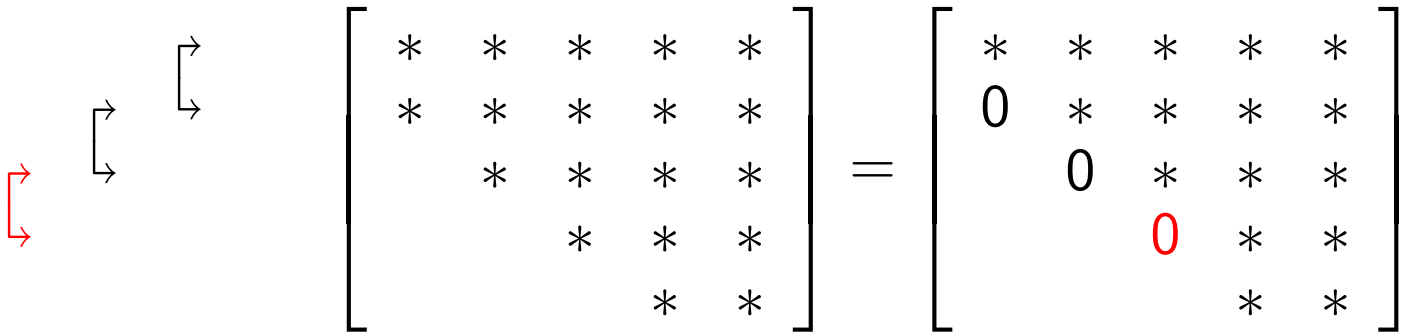

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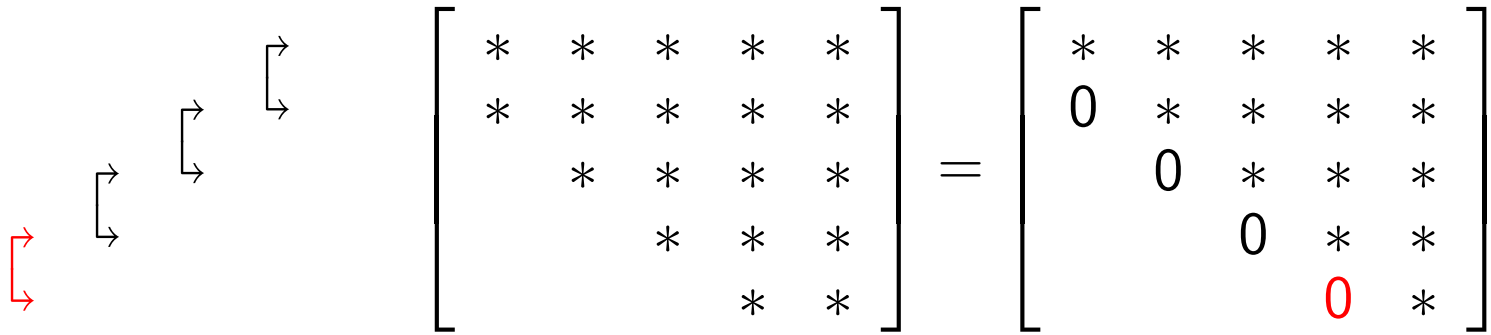

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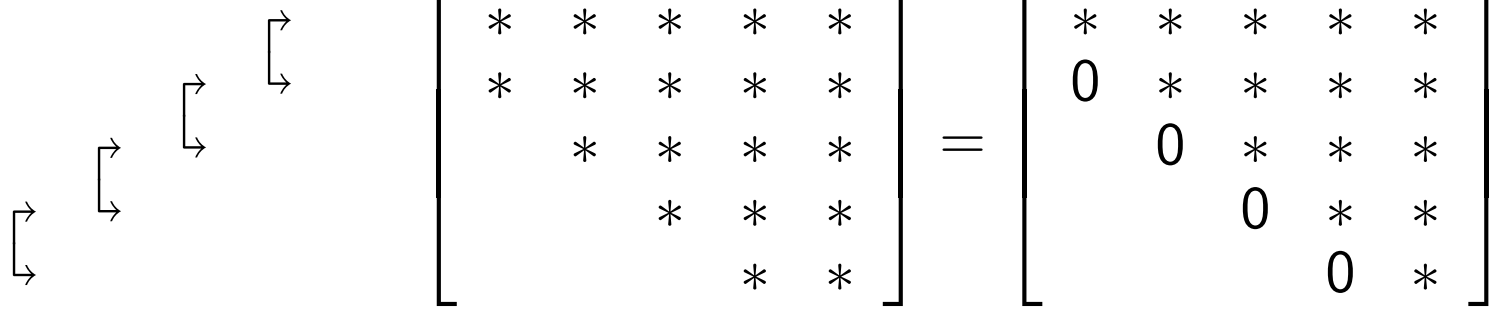

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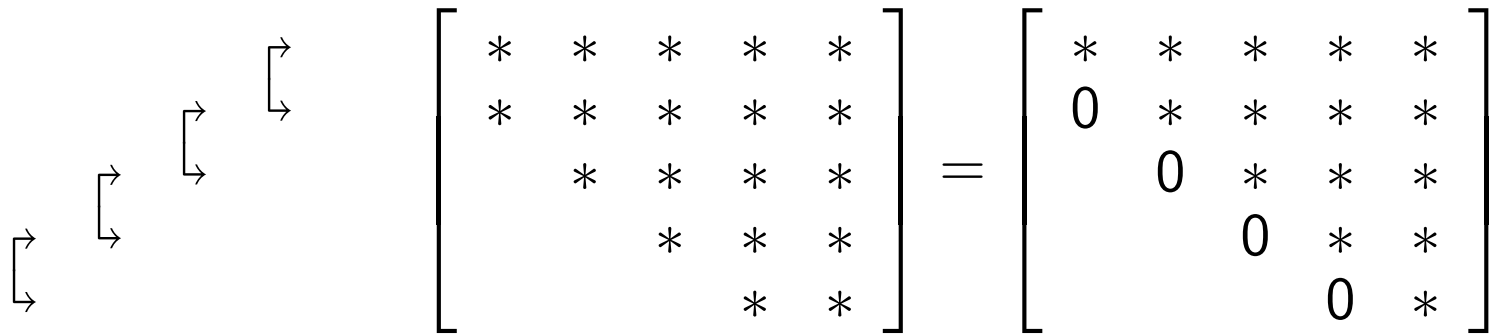

- **Def:** *Core Transformation*

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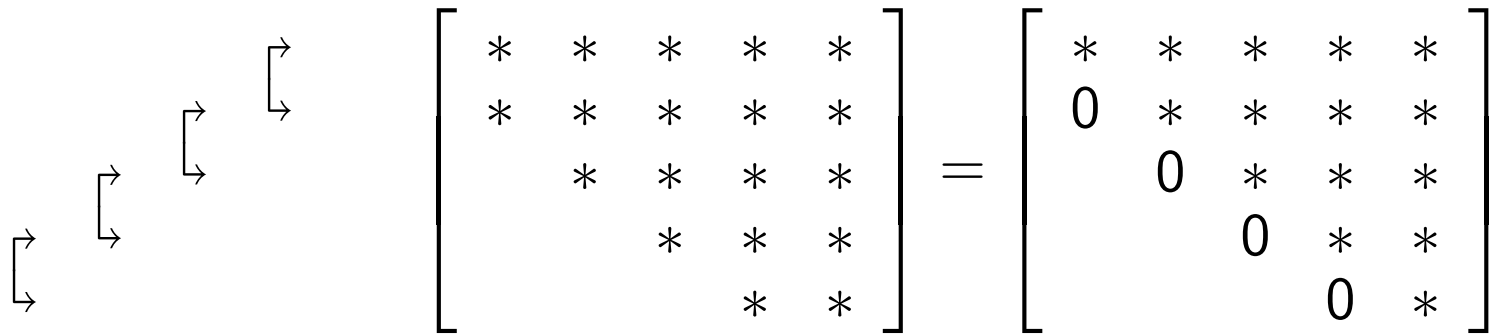

- **Def:** *Core Transformation*
- Now invert the core transformations to move them to the other side.

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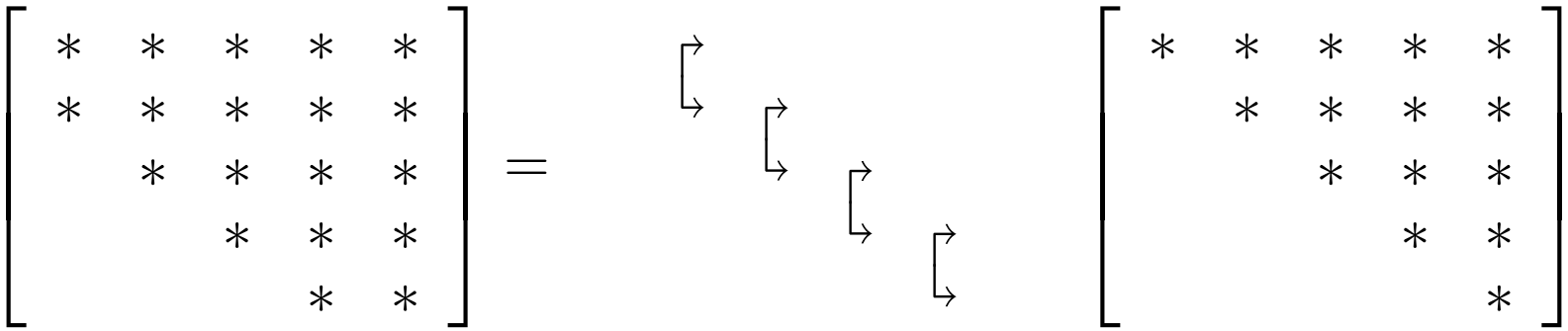

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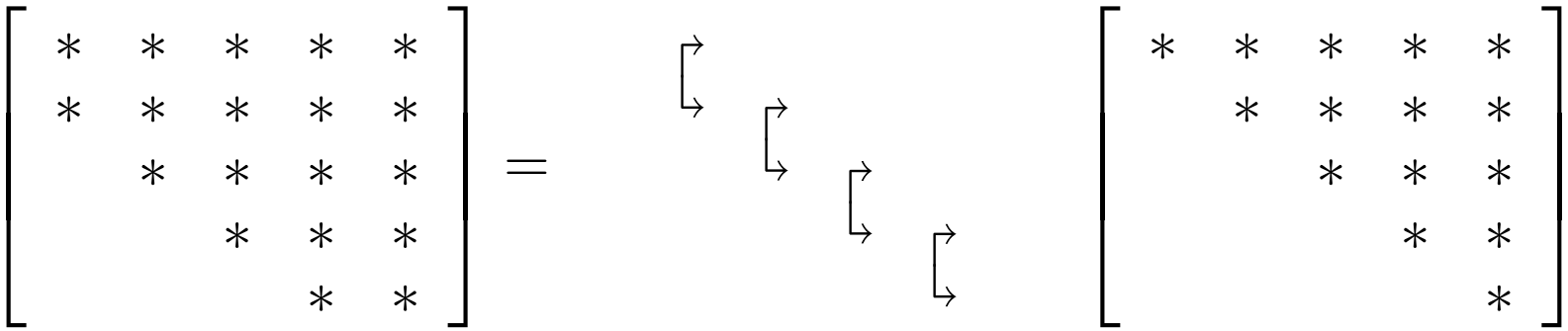

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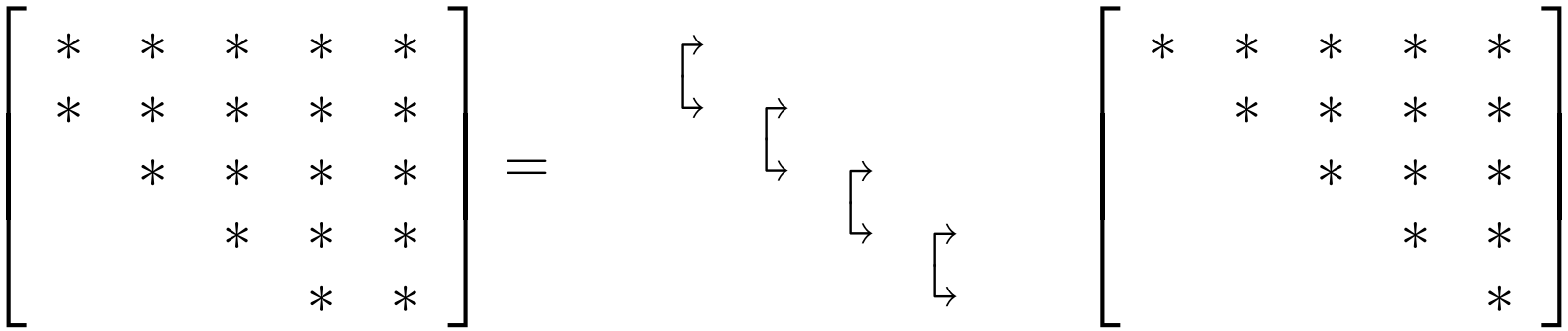

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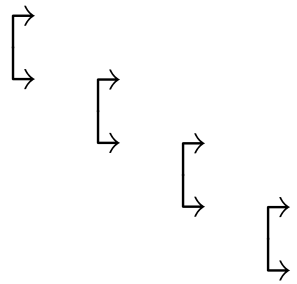
$$Q =$$ 

Q requires only $O(n)$ storage space.

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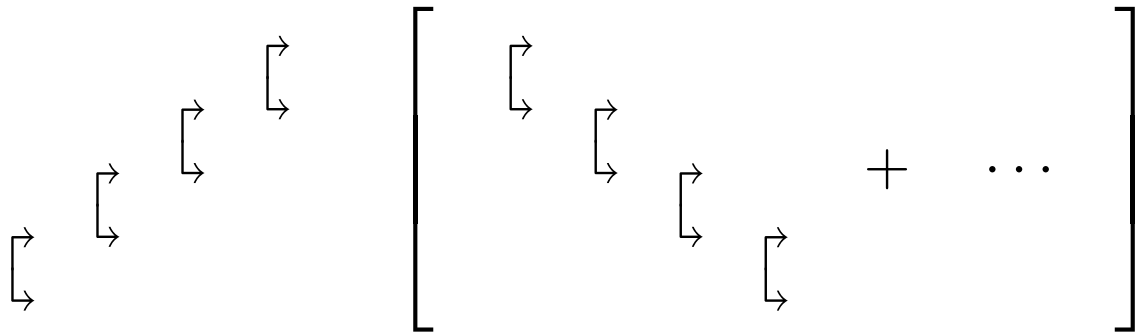
$\hat{R} =$ 

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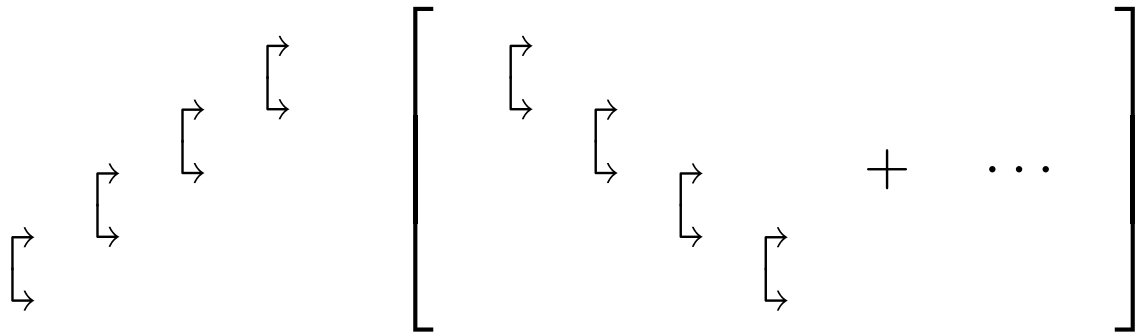
$\hat{R} =$ 

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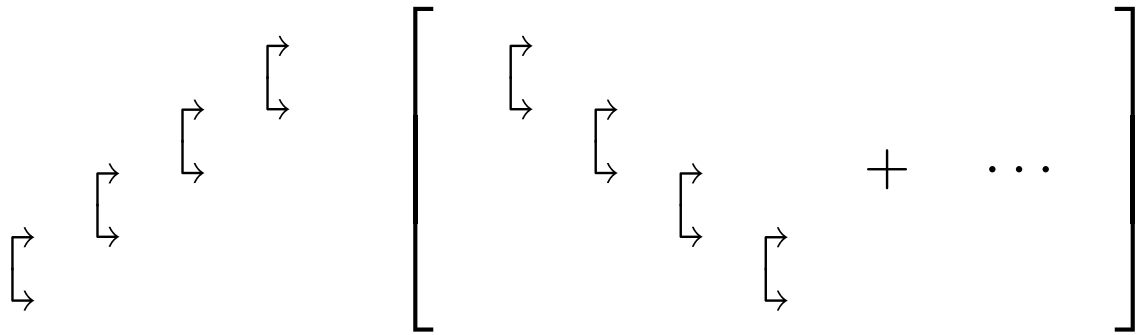
$\hat{R} =$ 

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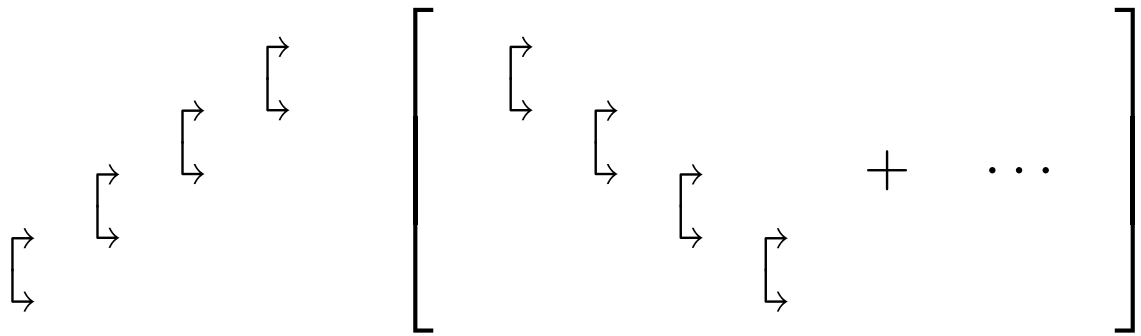
$\hat{R} =$ 

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Altogether we have

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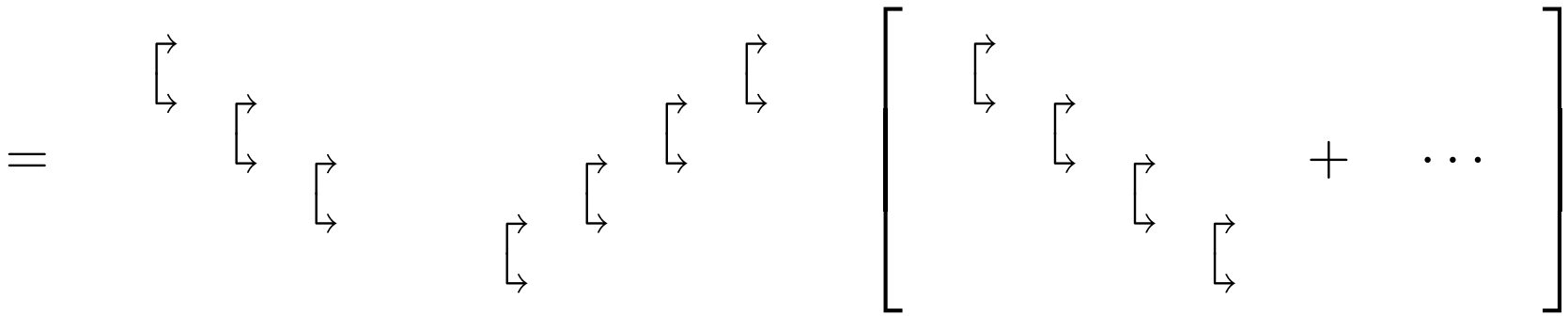

- A is stored entirely in terms of core transformations.

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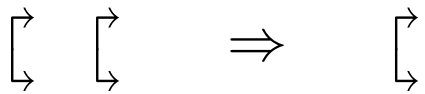

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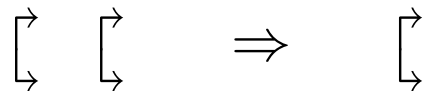
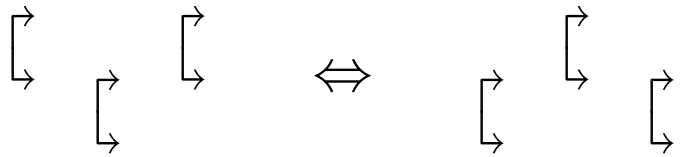
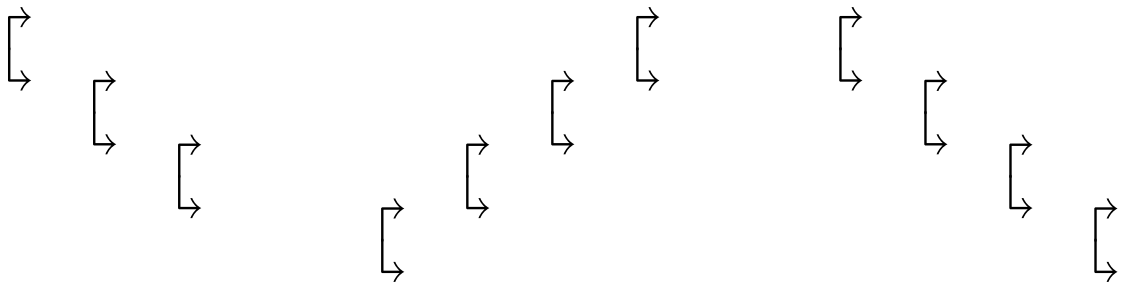


- Turnover (aka shift through, Givens swap, . . .)

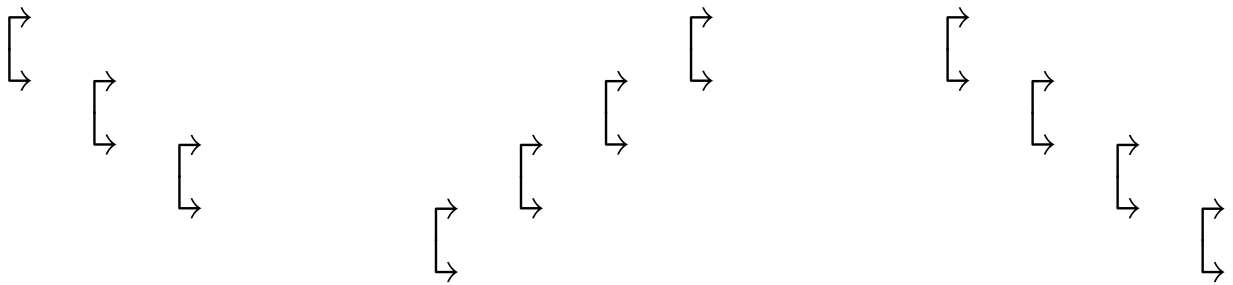


Francis Iteration (the core chase)

- ignoring rank-one part . . .

$A =$ 

The Core Chase



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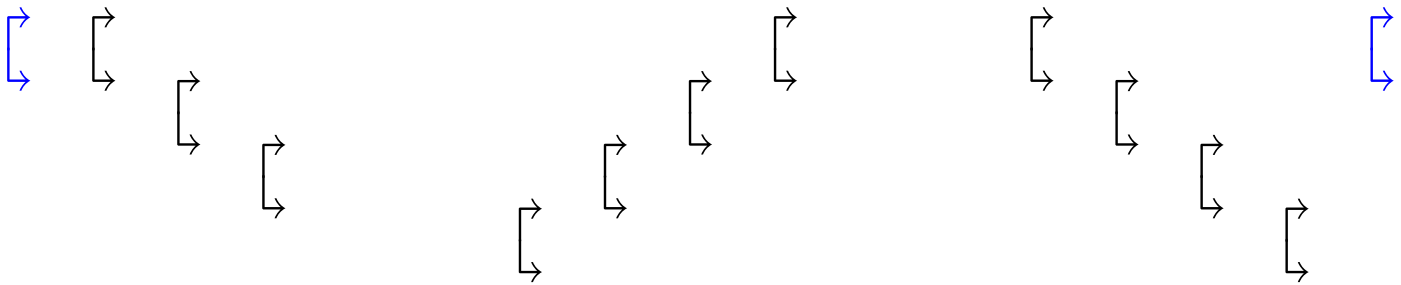

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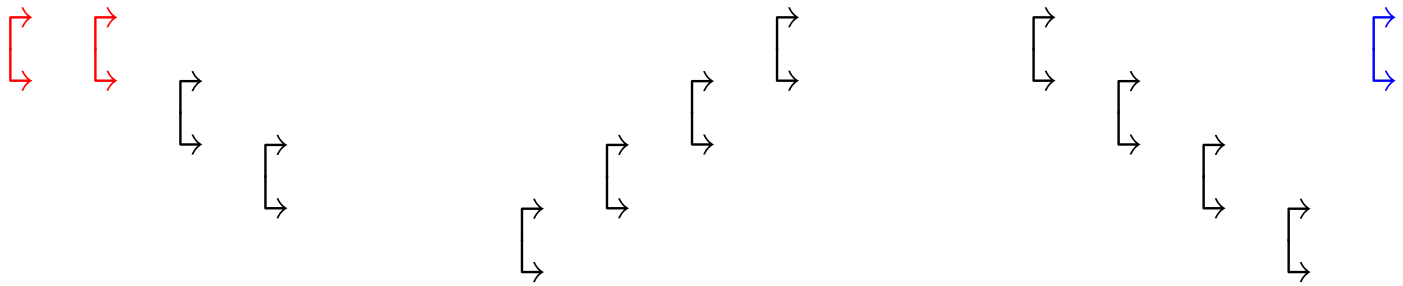

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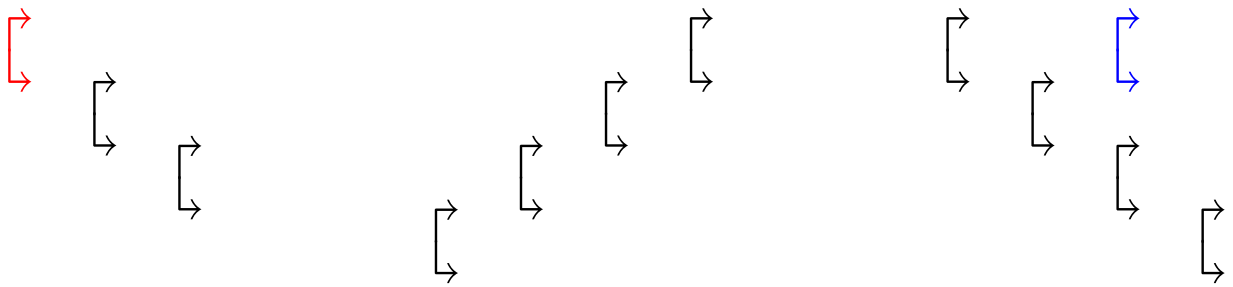

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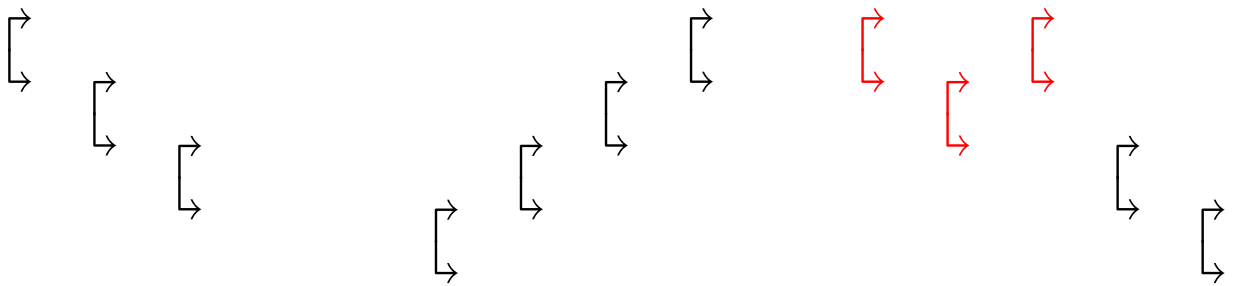

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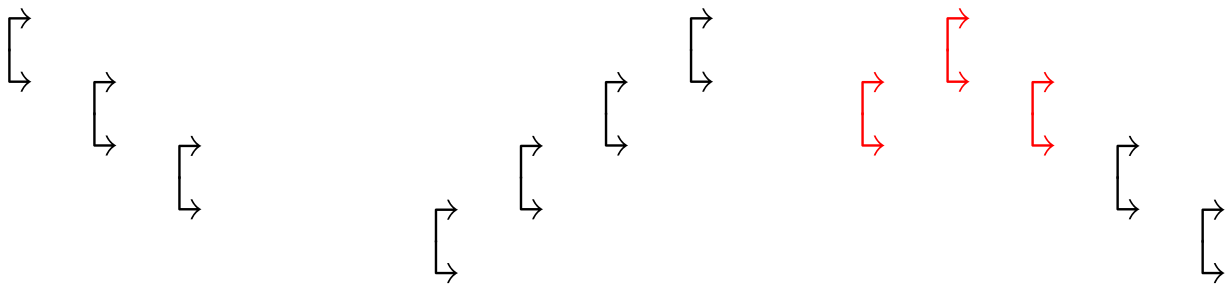

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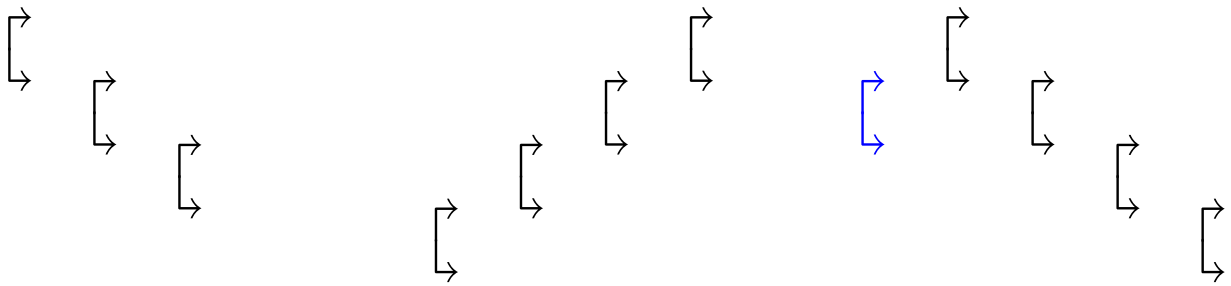

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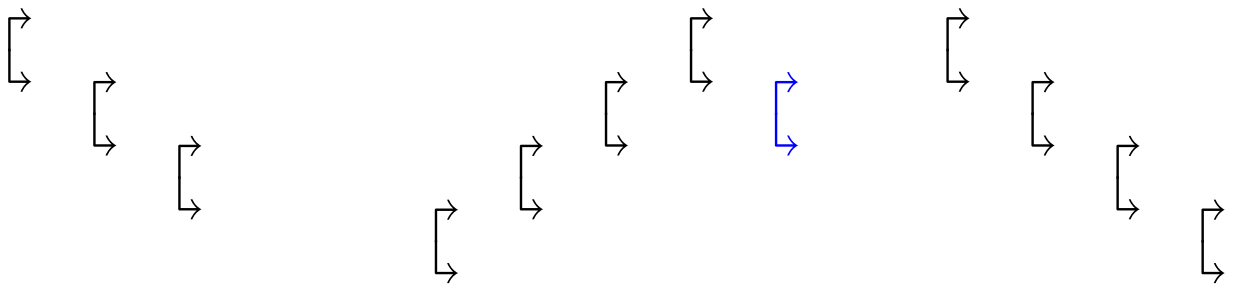

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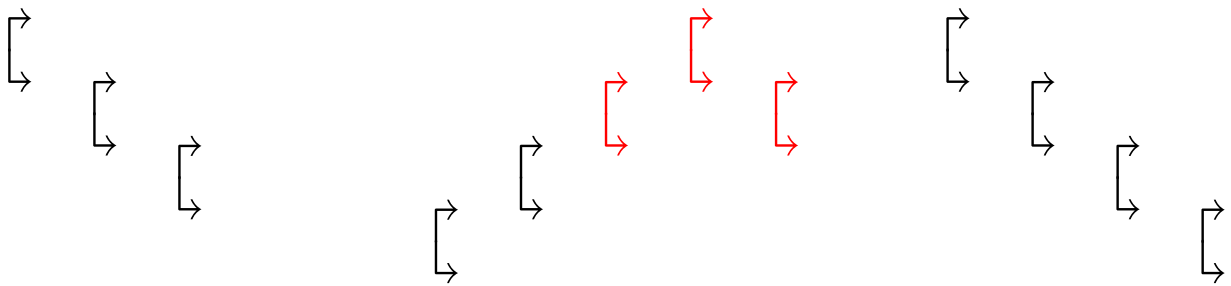

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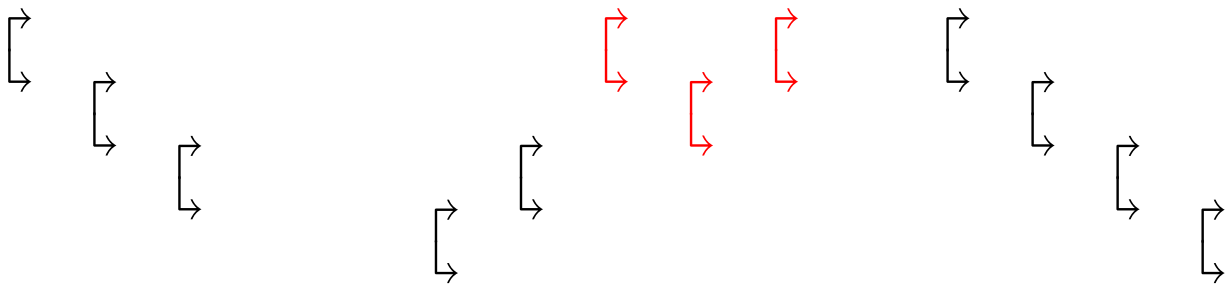

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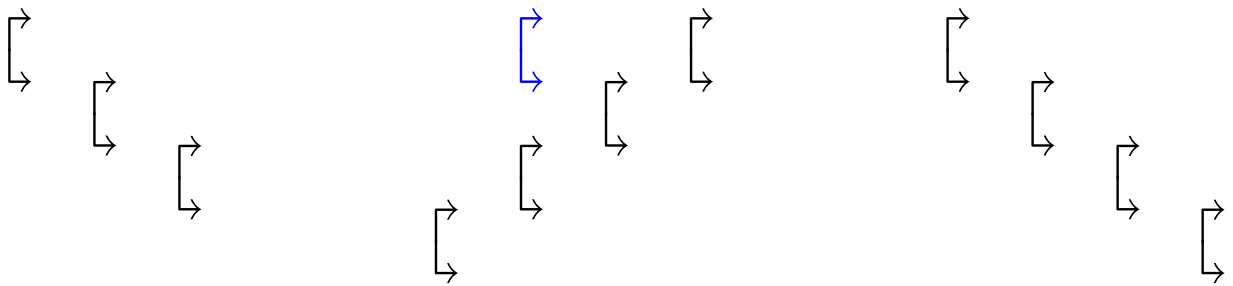

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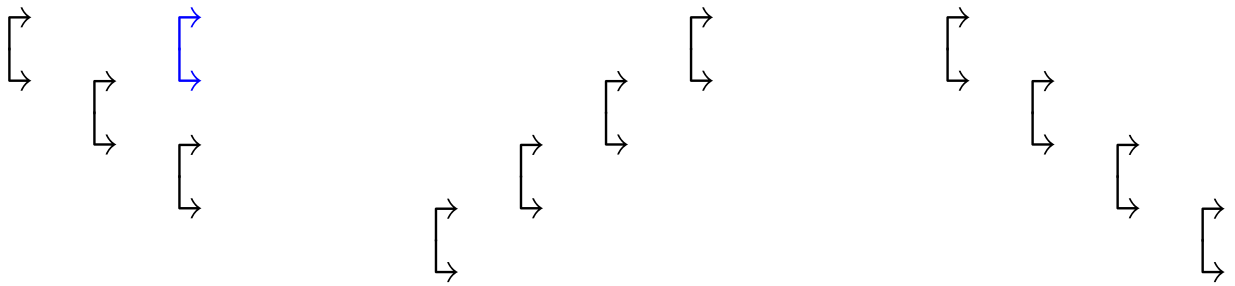

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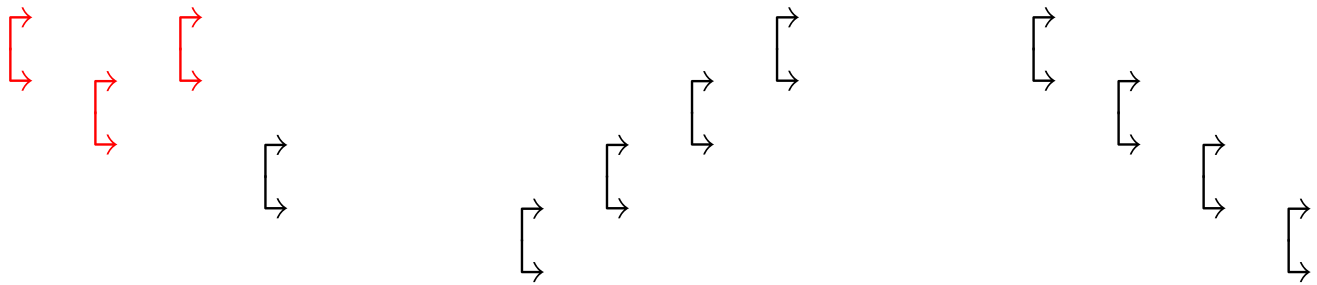

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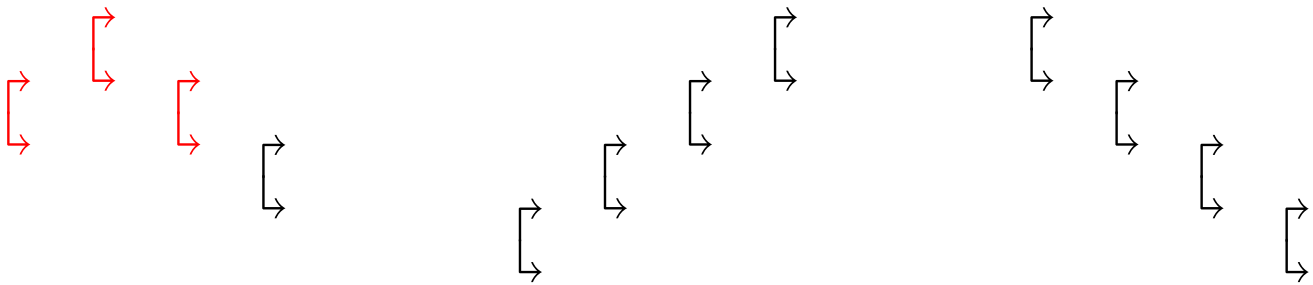

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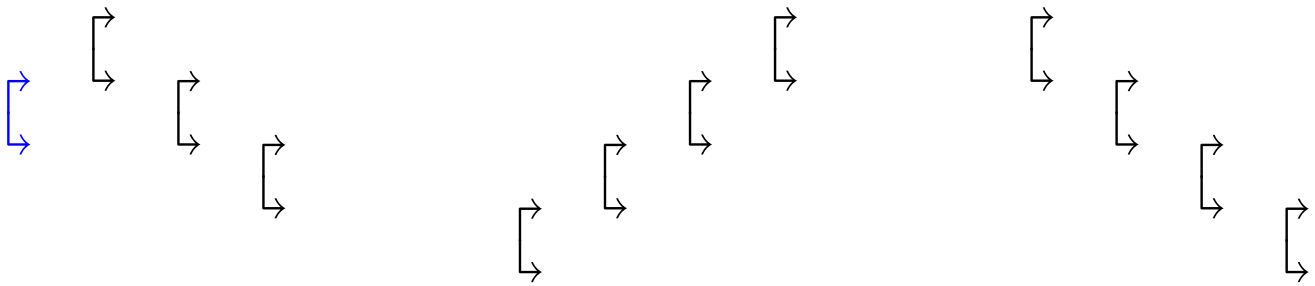

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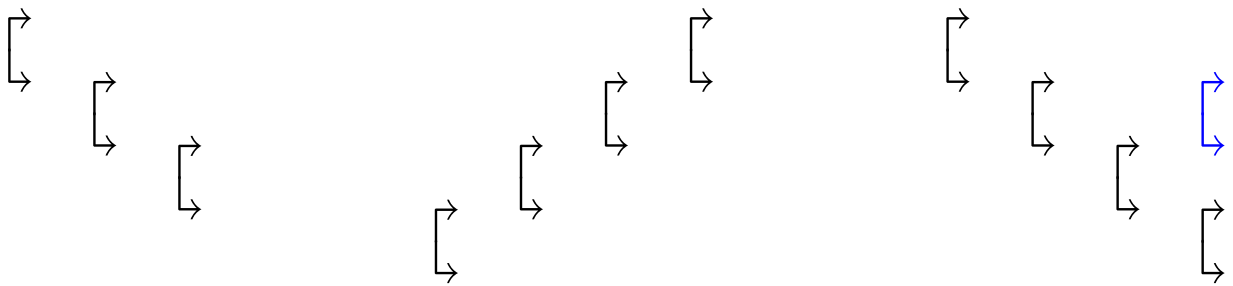

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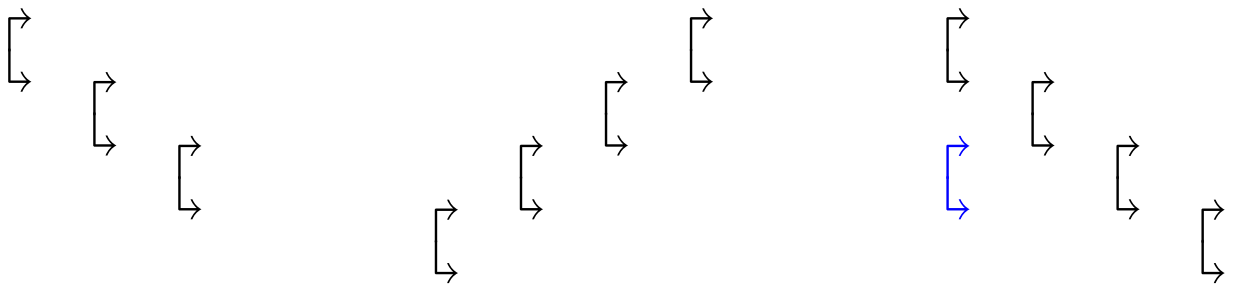

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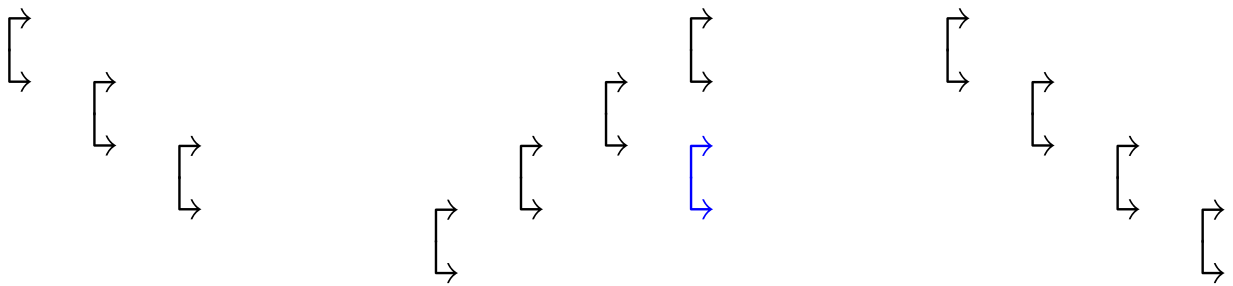

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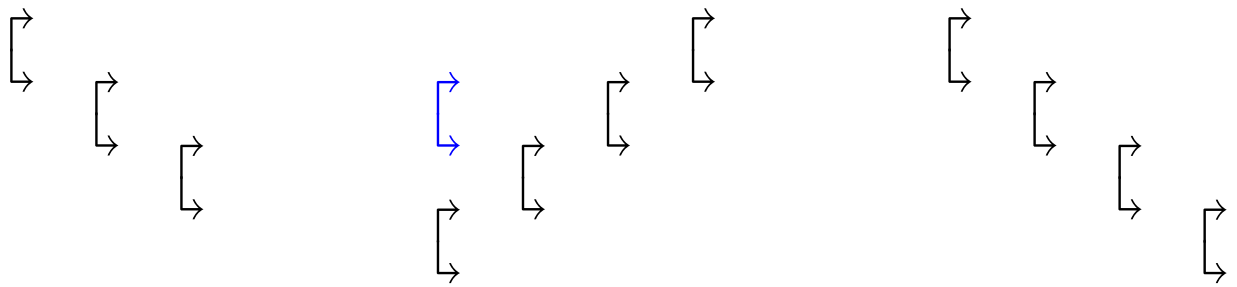

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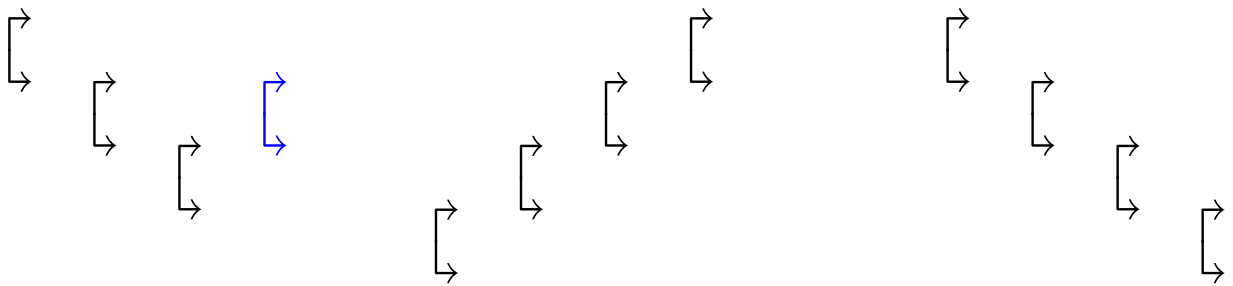

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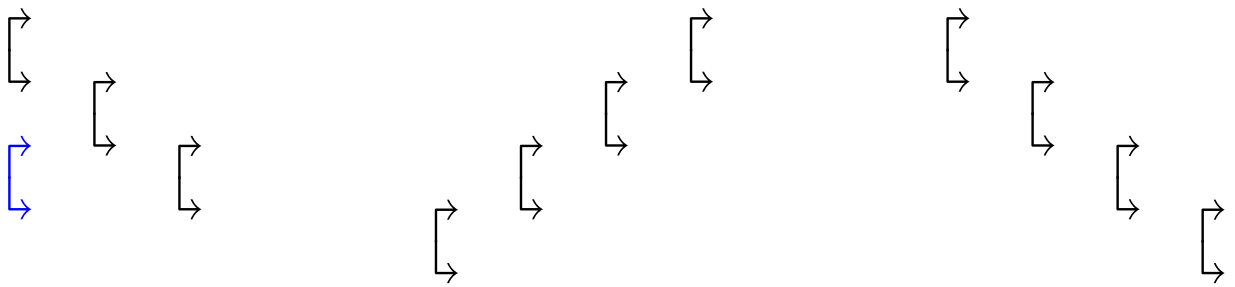

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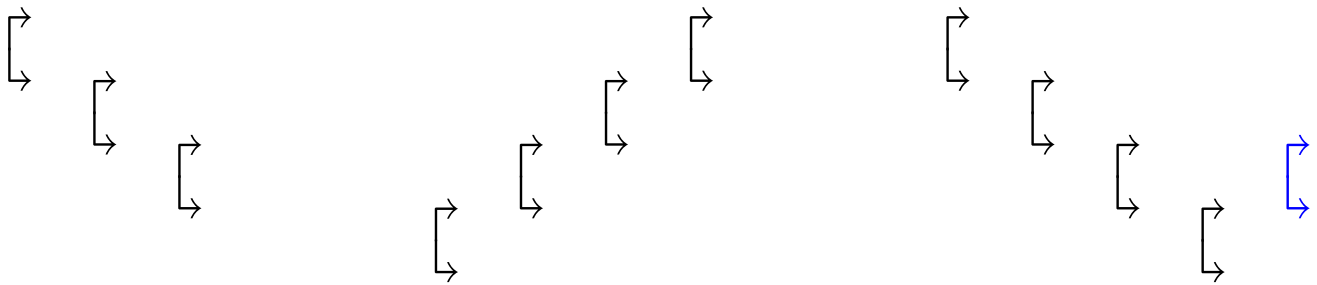

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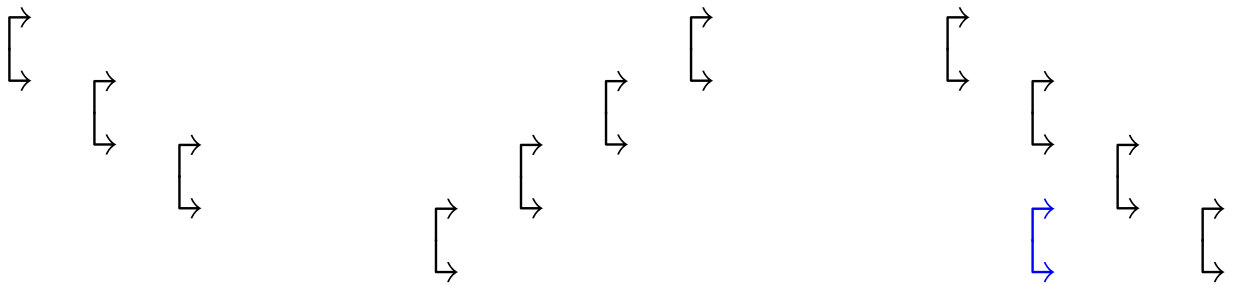

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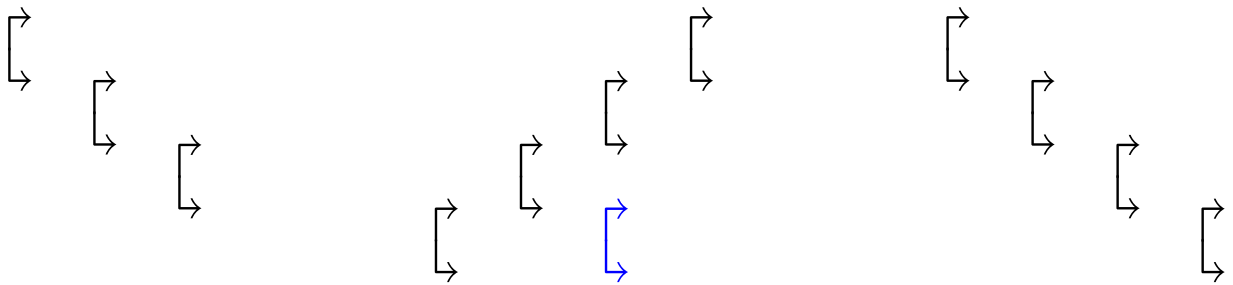

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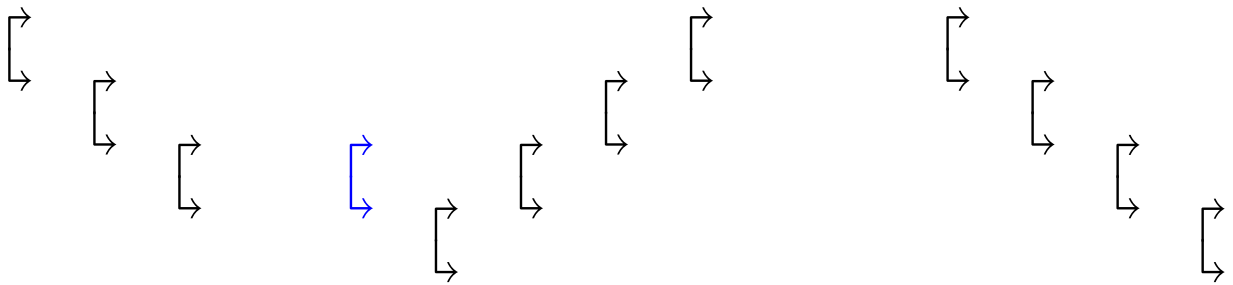

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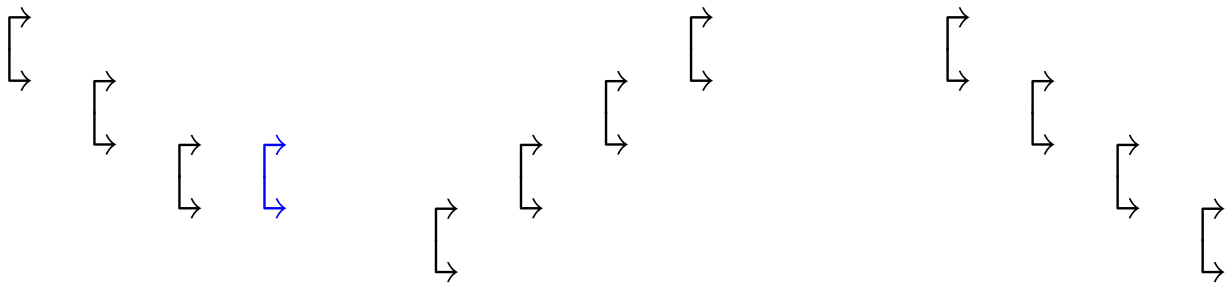

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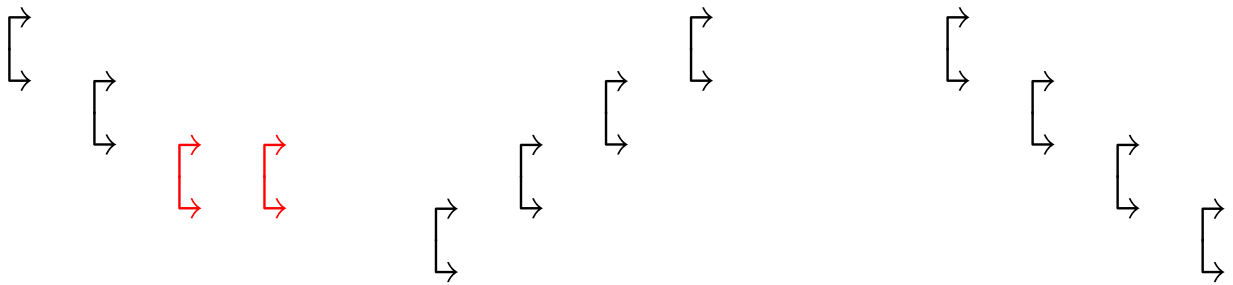

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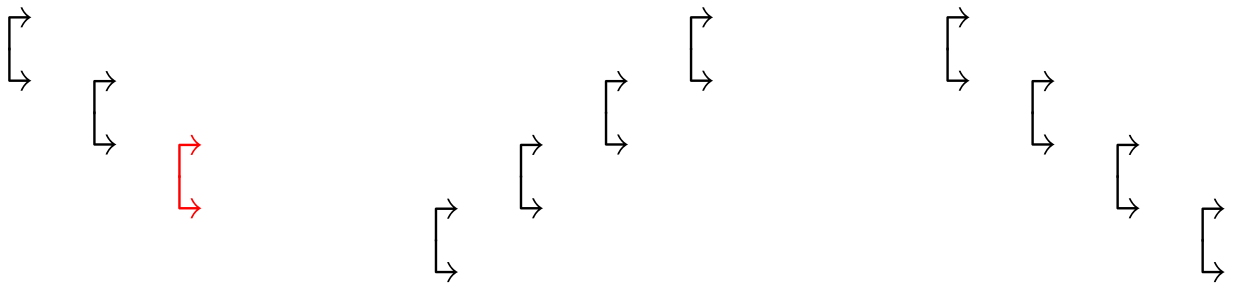

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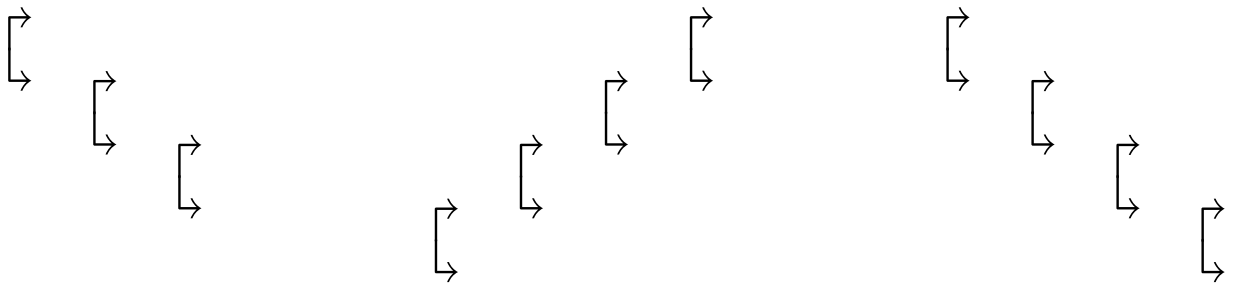


Done!

- iteration complete!

Done!

- iteration complete!
- Cost: $3n$ turnovers/iteration, so $O(n)$ flops/iteration.

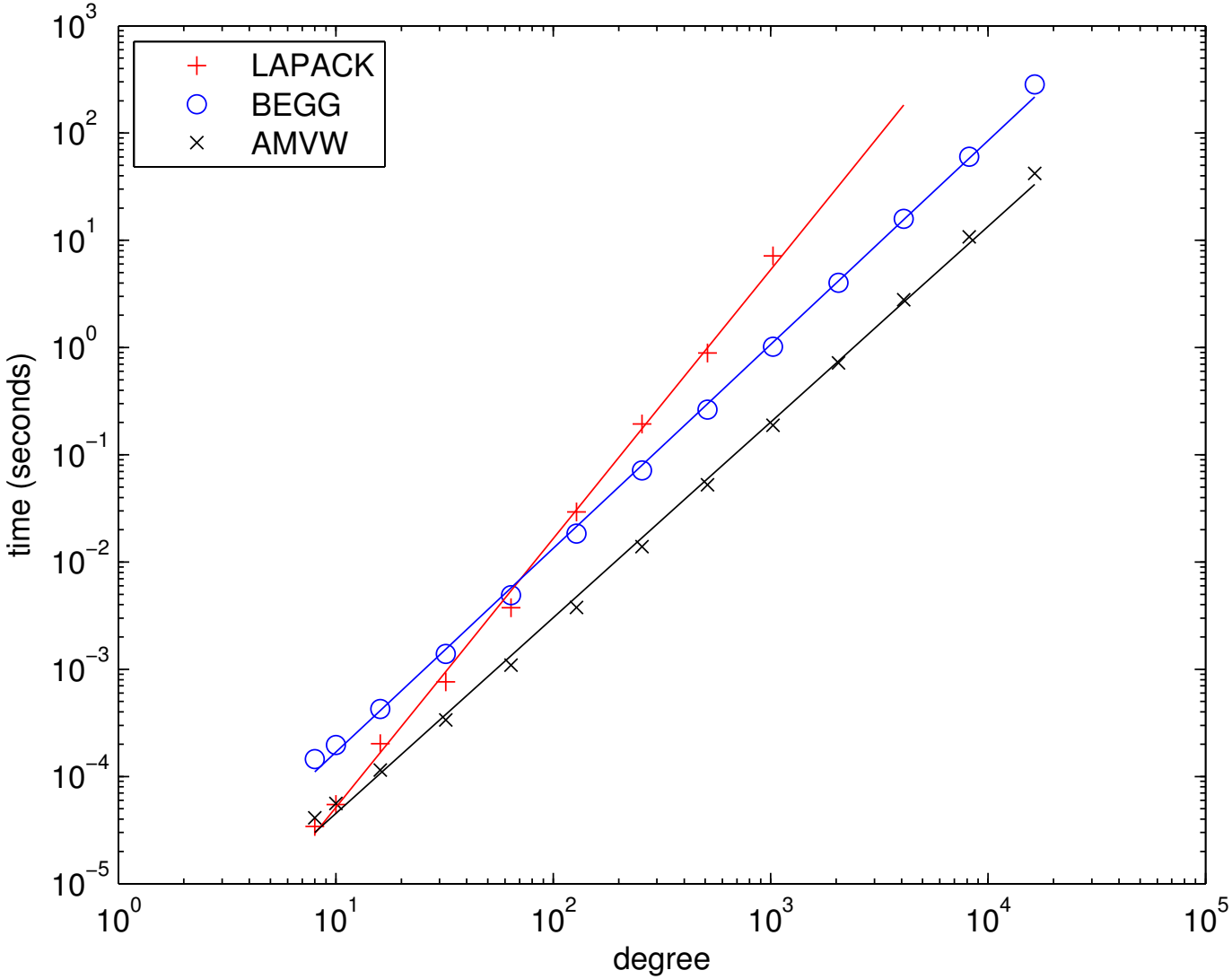
Done!

- iteration complete!
- Cost: $3n$ turnovers/iteration, so $O(n)$ flops/iteration.
- $O(n)$ iterations in all.

Done!

- iteration complete!
- Cost: $3n$ turnovers/iteration, so $O(n)$ flops/iteration.
- $O(n)$ iterations in all.
- Total flop count is $O(n^2)$.

Performance



Performance

At degree 1000

method	time
LAPACK	7.2
BEGG	1.2
AMVW	0.2

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Thank you for your attention.