Math 251 Quiz 8 Instructor: Sheng-Chi Liu

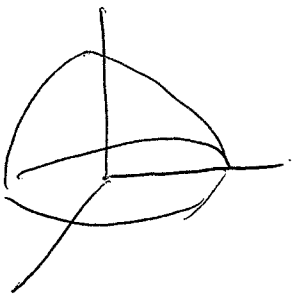
Name: Solution
UIN: _____

Show all your work! Credit will not be given without work.

1) (5 points) Evaluate

$$\int_{-3}^{3}\int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}}\int_{0}^{\sqrt{9-x^2-y^2}} z\sqrt{x^2+y^2+z^2}\,dz\,dy\,dx.$$

(Hint: use spherical coordinate, $dV = \rho^2 \sin\phi\, d\rho d\phi d\theta$.)



$$\int_0^{2\pi}\int_0^{\frac{\pi}{2}}\int_0^{3} \rho\cos\phi\cdot\rho\cdot\rho^2\sin\phi\, d\rho\, d\phi\, d\theta$$

$$= \int_0^{2\pi}\int_0^{\frac{\pi}{2}}\int_0^{3} \rho^4\cos\phi\sin\phi\, d\rho\, d\phi\, d\theta$$

$$= \left(\int_0^3 \rho^4\, d\rho\right)\left(\int_0^{\frac{\pi}{2}} \cos\phi\sin\phi\, d\phi\right)\left(\int_0^{2\pi} d\theta\right)$$

$$= \left[\frac{\rho^5}{5}\Big|_0^3\right]\left[\frac{\sin^2\phi}{2}\Big|_0^{\frac{\pi}{2}}\right]\cdot 2\pi$$

$$= \frac{3^5}{5}\cdot\frac{1}{2}\cdot 2\pi = \frac{3^5}{5}\pi$$

$$= \boxed{\frac{243}{5}\pi}$$

2)(5 points) Write the following equation in cylindrical coordinate.

$$x^2 + y^2 - z^2 = 16$$

$x = r\cos\theta$

$y = r\sin\theta$

$z = z$

$\Rightarrow$ $\boxed{r^2 - z^2 = 16}$