Math 251 Quiz 7 Instructor: Sheng-Chi Liu

Name: Solution
UIN: _____

Show all your work! Credit will not be given without work.

1) (5 points) Evaluate $\iiint_E yz dV$ where

$$E = \{(x, y, z) |\ 0 \le z \le 1,\ 0 \le y \le 2z,\ 0 \le x \le z + 2\}.$$

$$\iiint_E yz\, dV = \int_0^1 \int_0^{2z} \int_0^{z+2} yz\ dx\, dy\, dz$$

$$= \int_0^1 \int_0^{2z} yz\, x \Big|_{x=0}^{z+2} dy\, dz$$

$$= \int_0^1 \int_0^{2z} yz(z+2)\, dy\, dz$$

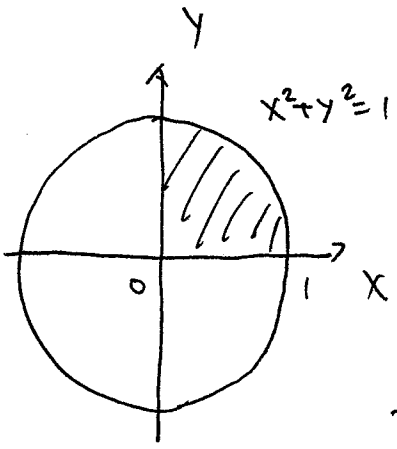
$$= \int_0^1 \frac{y^2}{2} z(z+2) \Big|_{y=0}^{2z} dz$$

$$= \int_0^1 2z^2 z(z+2)\, dz$$

$$= \int_0^1 2z^4 + 4z^3\, dz$$

$$= \frac{2}{5} z^5 + z^4 \Big|_0^1 = \frac{2}{5} + 1 = \boxed{\frac{7}{5}}$$

2)(5 points) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \cos(x^2+y^2)dydx$



$y = \sqrt{1-x^2}$

$\Rightarrow x^2+y^2 = 1$

Use Polar coord. $x = r\cos\theta$, $y = r\sin\theta$

The integral $= \int_0^{\frac{\pi}{2}} \int_0^1 (\cos r^2)\, r\, dr\, d\theta$

$= \int_0^{\frac{\pi}{2}} \frac{1}{2}\sin r^2 \Big|_{r=0}^{1} d\theta$

$= \int_0^{\frac{\pi}{2}} \frac{1}{2}\sin 1 \, d\theta$

$= \frac{1}{2}\sin 1 \cdot \frac{\pi}{2} = \boxed{\frac{\pi}{4}\sin 1}$