Math 251 **Quiz 5** **Instructor: Sheng-Chi Liu**

Name: Solution

UIN: ______________

Show all your work! Credit will not be given without work.

1) (5 points) Given $f(x, y) = x^3 - 4x^2y + y^2$ and $\mathbf{u} = < \frac{-1}{\sqrt{5}}, \frac{2}{\sqrt{5}} >$. Find $D_{\mathbf{u}}f(0, -1)$.

$$f_x = 3x^2 - 8xy$$

$$f_y = -4x^2 + 2y$$

$$\nabla f(0, -1) = \langle 0, -2 \rangle$$

$$D_u f(0, -1) = \langle 0, -2 \rangle \cdot \langle -\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}} \rangle$$

$$= \boxed{\frac{-4}{\sqrt{5}}}$$

2) (5 points) Find the local maximum and local minimum and saddle points of

$$f(x,y) = x^3 - 3xy + y^3.$$

$f_x = 3x^2 - 3y$

$f_y = -3x + 3y^2$

Solve $\begin{cases} 3x^2 - 3y = 0 \\ -3x + 3y^2 = 0 \end{cases} \Rightarrow \begin{cases} y = x^2 \\ y^2 - x = 0 \end{cases}$

$\Rightarrow x^4 - x = 0 \Rightarrow x(x^3 - 1) = 0$

$\Rightarrow x(x-1)(x^2 + x + 1) = 0$

$\Rightarrow \begin{cases} x = 0 \\ y = 0 \end{cases}$ or $\begin{cases} x = 1 \\ y = 1 \end{cases}$

$$D = \begin{vmatrix} 6x & -3 \\ -3 & 6y \end{vmatrix} = 36xy - 9$$

① $D(0.0) = -9 < 0 \Rightarrow$ $\boxed{(0.0) \text{ is a saddle point}}$

② $D(1.1) = 36 - 9 = 27 > 0$

$f_{xx}(1.1) = 6 \cdot 1 = 6 > 0$

$\boxed{f(1.1) = 1 - 3 + 1 = -1 \text{ is a local min.}}$