Math 251 Quiz 4 Instructor: Sheng-Chi Liu

Name: Solution

UIN: _____

Show all your work! Credit will not be given without work.

1) Find the limit, if it exists, or show that the limit does not exist.

a) (2 points) $\lim_{(x,y)\to(\pi,\pi)} x\sin(\frac{x+y}{4})$.

b) (3 points) $\lim_{(x,y)\to(0,0)} \frac{2x^2y}{x^4+y^2}$.

a) $\lim_{(x,y)\to(\pi,\pi)} x \sin\left(\frac{x+y}{4}\right) = \pi \sin\frac{\pi}{2} = \boxed{\pi}$

b) $\lim_{(x,y)\to(0,0)} \frac{2x^2y}{x^4+y^2} = \lim_{(x,y)\to(0,0)} \frac{0}{y^2} = 0$

$x=0$ $\quad x=0$

~~$\lim_{(x,y)\to(0,0)} \frac{2x^2y}{x^4+y^2} = \lim_{(x,y)\to(0,0)} \frac{2x^2\cdot x^2}{x^4+x^4} = \lim_{(x,y)\to(0,0)} \frac{2x^4}{2x^4} = 1$~~

$y=x^2$ $\quad y=x^2$ $\quad y=x^2$

So $\lim_{(x,y)\to(0,0)} \frac{2x^2y}{x^4+y^2}$ does NOT exist

2) (5 points) If $z = 5x^2 + y^2$ and (x, y) changes from $(1, 2)$ to $(1.05, 2.1)$, find dz.

$$dz = f_x(x,y)\,dx + f_y(x,y)\,dy$$

$$= 10x\,dx + 2y\,dy$$

$x = 1$, $y = 2$

$dx = 1.05 - 1 = 0.05$

$dy = 2.1 - 2 = 0.1$

Hence

$$dz = 10 \times 1 \times 0.05 + 2 \times 2 \times 0.1$$

$$= 0.5 + 0.4 = \boxed{0.9}$$