Math 251 **Quiz 3** **Instructor: Sheng-Chi Liu**

Name: Solution

UIN: ______________

Show all your work! Credit will not be given without work.

Given the curve $\mathbf{r}(t) = < \sin 4t,\ 3t,\ \cos 4t >$.

1) (3 points) Find the unit tangent vector $\mathbf{T}(t)$.
2) (2 points) Find the unit normal vector $\mathbf{N}(t)$.
3) (2 points) Find the binormal vector $\mathbf{B}(t)$.
4) (3 points) Find the arc length from $t = 0$ to $t = 2$.

1) $r'(t) = \langle 4\cos 4t,\ 3,\ -4\sin 4t \rangle$

$|r'(t)| = \sqrt{16\cos^2 4t + 9 + 16\sin^2 4t} = \sqrt{25} = 5$

$T(t) = \dfrac{r'(t)}{|r'(t)|} = \langle \frac{4}{5}\cos 4t,\ \frac{3}{5},\ -\frac{4}{5}\sin 4t \rangle$

2) $T'(t) = \langle -\frac{16}{5}\sin 4t,\ 0,\ -\frac{16}{5}\cos 4t \rangle$

$|T'(t)| = \sqrt{(\frac{16}{5})^2\sin^2 4t + (\frac{16}{5})^2\cos^2 4t} = \frac{16}{5}$

$N(t) = \dfrac{T'(t)}{|T'(t)|} = \langle -\sin 4t,\ 0,\ -\cos 4t \rangle$

3) $B(t) = T(t) \times N(t) = -\frac{3}{5}\cos 4t\,\vec{i} + \frac{4}{5}\vec{j} + \frac{3}{5}\sin 4t\,\vec{k}$

4) $L = \int_0^2 |r'(t)|\,dt = \int_0^2 5\,dt = 10$