Exam 1 Solution Math 251 Fall 2011

1 A, 2 B, 3 B, 4 D, 5 C, 6 A, 7 C, 8 B, 9 D

10. ① along $x=2$

$$\lim_{\substack{(x,y)\to(2,0)\\ x=2}} \frac{xy-2y}{x^2+y^2-4x+4} = \lim_{\substack{(x,y)\to(2,0)\\ x=2}} \frac{2y-2y}{4+y^2-8+4} = 0$$

② along $y=x-2$

$$\lim_{\substack{(x,y)\to(2,0)\\ y=x-2}} \frac{xy-2y}{x^2+y^2-4x+4} = \lim_{\substack{(x,y)\to(2,0)\\ y=x-2}} \frac{x(x-2)-2(x-2)}{x^2+(x-2)^2-4x+4}$$

$$= \lim_{\substack{(x,y)\to(2,0)\\ y=x-2}} \frac{x^2-4x+4}{2(x^2-4x+4)} = \frac{1}{2}$$

Hence $\lim_{(x,y)\to(2,0)} \frac{xy-2y}{x^2+y^2-4x+4}$ does NOT exist.

11. $$\frac{\partial g}{\partial t} = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial t}$$

$$= 2tu^4 \sin(t\ln u) + (tu^2)^2 \cos(t\ln u)\cdot \ln u$$

$$\frac{\partial g}{\partial u} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial u}$$

$$= 4t^2u^3 \sin(t\ln u) + t^3u^3 \cos(t\ln u)$$

12 a) $dz = e^y\,dx + xe^y\,dy$

b) $f(1.99, 0.02) \approx f(2,0) + dz = 2.03$

[Here $x=0$, $dx = 1.99-2 = -0.01$
$y=0$, $dy = 0.02-0 = 0.02$
So $dz = 0.03$]