On Strongly Infinitely Divisible Matrices

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Roots of a nonnegative matrix (if they exist) may or may not be nonnegative. The matrix exponential $A = e^B$ of an (essentially) nonnegative matrix $B$ is indeed a nonnegative invertible matrix and all of whose nonnegative powers $A^t = e^{tB}$ ($t \geq 0$) are clearly also nonnegative. The converse is also true, as shown recently by Van-Brunt. If $A$ is an invertible nonnegative matrix all of whose roots exist and are also nonnegative, there exists a nonnegative matrix $B$ such that $A = e^B$. We refer to such an $A$ as a strongly infinitely divisible matrix ($A \in \text{SIDM}$). An inverse $M$-matrix is a particular example of an SIDM. Inverse $M$-matrices play an important role in this study.

Properties and connections of SIDM’s to other matrix classes is still under consideration. In my talk we will discuss certain operations that leave SIDM invariant, examine submatrices of SIDM’s, discover an intimate connection of SIDM’s and their roots to P-matrices and eventually nonnegative matrices. Further, we will discuss Hadamard/Kronecker products of SIDM’s, monotonicity of roots of SIDM’s and eigenvalues of SIDM’s.