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Title: Gap Principle of Divisibility Sequences of Polynomials

Abstract: Let $f \in \mathbb{Z}[x]$ and $\ell \in \mathbb{N}$. Consider the set of all $(a_0, a_1, \ldots, a_\ell) \in \mathbb{N}^{\ell+1}$ with $a_i < a_{i+1}$ and $f(a_i) \mid f(a_{i+1})$ for all $0 \leq i \leq \ell - 1$. We say that f satisfies the gap principle of order ℓ if $\lim a_\ell/a_0 = \infty$ as $a_0 \to \infty$. We also define the gap order of $f(x)$ to be the smallest positive integer ℓ such that $f(x)$ satisfies the gap principle of order ℓ. If such ℓ does not exist, we say that $f(x)$ does not satisfy the gap principle. In this talk, we will discuss a proof of the conjecture by Chan, Choi and Lam that $f(x)$ does not satisfy the gap principle if and only if $f(x)$ is in the form of $f(x) = A(Bx + C)^n$ for some $A, B, C \in \mathbb{Z}$. Moreover, we completely determine the gap order of any polynomial. We will show that if $f(x)$ is not in the form of $A(Bx + C)^n$, then $f(x)$ has gap order 2 if $f(x)$ is a quadratic polynomial or a power of a quadratic polynomial; and has gap order 1 otherwise.