PASCAL EIGENSPACES AND INVARIANT SEQUENCES OF THE FIRST OR SECOND KIND

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ABSTRACT

The binomial inversion formula states that for sequences $\{a_n\}, \{b_n\}$ $(n = 0, 1, \ldots)$,

$$a_n = \sum_{k=0}^{n} \binom{n}{k} (-1)^k b_k \quad \text{if and only if} \quad b_n = \sum_{k=0}^{n} \binom{n}{k} (-1)^k a_k. \tag{1.1}$$

This result motivated Sun to investigate sequences $\{a_n\}$ such that

$$(-1)^{s-1} a_n = \sum_{k=0}^{n} \binom{n}{k} (-1)^k a_k; \quad s = 1 \text{ or } s = 2. \tag{1.2}$$

We refer to such sequences $\{a_n\}$ as *invariant* (when $s = 1$) or *inverse invariant* (when $s = 2$) *sequences of the first kind*. It can be shown that $\{a_n\}$ is inverse invariant sequence of the first kind if and only if $a_0 = 0$ and $\{\frac{a_{n+1}}{n+1}\}$ or $\{n\, a_{n-1}\}$ is an invariant sequence of the first kind. Several examples of invariant sequences of the first kind can be found in his paper, including

$$\{\frac{1}{2^n}\}, \ \{nF_{n-1}\}, \ \{L_n\}, \ \{(-1)B_n\},$$

where $\{F_n\}$, $\{L_n\}$, $\{B_n\}$ are the Fibonacci sequence, Lucas sequence and Bernoulli numbers, respectively.

In this paper, we will establish the 'modified' binomial inversion formula

$$a_n = \sum_{k=n}^{\infty} \binom{k}{n} (-1)^k b_k \quad \text{if and only if} \quad b_n = \sum_{k=n}^{\infty} \binom{k}{n} (-1)^k a_k. \tag{1.3}$$

Motivated by (1.3) we will consider sequences $\{a_n\}$ $(n = 0, 1, \ldots)$ such that

$$(-1)^{s-1} a_n = \sum_{k=n}^{\infty} \binom{k}{n} (-1)^k a_k; \quad s = 1 \text{ or } s = 2. \tag{1.4}$$

We refer to such sequences $\{a_n\}$ as *invariant* (when $s = 1$) and *inverse invariant* (when $s = 2$) *sequences of the second kind*.

Naturally arising is the question of existence and relevance of (inverse) invariant sequences of the second kind, as well as the problem of characterizing such sequences and examining their relationship to their counterparts of the first kind. To address these issues, we will follow the ideas and connections of invariant sequences to the eigenspaces of Pascal-type matrices developed in Choi et al. Specifically, we will associate (inverse) invariant sequences of the first and second kind to the eigenspaces of matrices constructed via the Pascal matrix. In doing so, we will continue and extend the study of such sequences via matrix theory, allowing us to also discover new relationships among such sequences.