Directions:

- Read and follow all directions carefully.
- You must show work or justify your reasoning with meaningful evidence, to receive full credit.
- Simplify completely whenever possible, unless otherwise stated.
- Box or circle your final answer.
- Label units appropriately.
- If you have any questions or need clarification on directions, please raise your hand and the proctor will come help you.
- No calculators, cell phones, or other electronic devices allowed during the exam.

1. Compute the derivative of the following functions. Do not simplify.

   a. (4 points) \( f(x) = x^2e^{-x} \)
Problem 1 continued

b. (4 points)
\[ g(x) = \ln (2x^3 - 7x + \pi^2) \]

c. (4 points)
\[ h(x) = \frac{x^2 + e^{3x}}{3x - 2} \]

d. (4 points)
\[ k(x) = 6^x + \log_2(x) \]
2. Let $h(x) = f(g(x))$ where $f(x) = \ln(x^2 + 3)$ and $g(x) = 2x + 1$.

   a. (2 points) Find $h(x)$

   b. (2 points) Find $h'(x)$

   c. (2 points) Evaluate $h'(1)$

3. The price – demand equation is given by $f(p) = 1300 - p^2$. The elasticity of demand is defined by

   $$E(p) = \frac{-p f'(p)}{f(p)}$$

   a. (3 points) Find $E(p)$ for the $f(p)$ given above.

   b. (2 points) Evaluate $E(30)$ and describe the type of elasticity at the price $p = $30.

   c. (2 points) Would it make sense to increase the current $30 price? Circle the appropriate answer. No work needed.

   YES       NO
4. (10 points) Find the equation of the tangent line(s) to the graph of \( y = y(x) \), where \( xy^2 - x + 18y + 30 = 0 \), when \( x = 2 \). Treat \( y \) as a function of \( x \).
5. (6 points) The area of an oil spill $t$ hours after the accident can be approximated by the area of a circle with radius $r$, $A = \pi r^2$. The radius of the oil spill is increasing at 2 meters per second when the radius of the oil spill is 100 meters. Find the rate of change of the area of the oil spill, $\frac{dA}{dt}$.

6. (2 points each) Write “TRUE” next to the statement if it is a correct statement and write “FALSE” next to the statement if it is an incorrect statement. No work needed.

a. The demand function $f(p)$ is a decreasing function, where $p$ is the price of an item.

b. L’Hopital’s Rule can be used to evaluate any limit.

c. If $0 < E(p) < 1$, then an increase in price increases revenue.

d. If $A = Pe^{rt}$, then the time it takes for the principle, $P$, to double when it is invested at rate $r$ compounded continuously is $t = \frac{\ln(2)}{r}$.

e. If $f(x)$ is a continuous function on $(a, b)$, $a < c < b$, and $f(c)$ is a local maximum, then $f(c) = 0$ or $f(c) \ DNE.$
7. (5 points each) Evaluate the following limits. Use L’Hopital’s rule only when applicable and indicate when you do. Fully justify each step.

a. \[
\lim_{x \to \infty} \frac{\ln(x + 4)}{x + 4}
\]

b. \[
\lim_{x \to 1} \frac{\ln x}{1 - 2x}
\]

c. \[
\lim_{x \to 0} \frac{e^{2\pi x} - 1}{x}
\]
8. Use the function \( f(x) = 2x^4 - 3x^3 \) to determine the following.

a. (2 points) Find \( f'(x) \).

b. (2 points) Find the critical values for \( f(x) \).

c. (4 points) Make a sign chart for \( f'(x) \).

d. (2 points) Give the x-coordinate of the local extrema of \( f(x) \). Circle the type of extrema that you listed.

<table>
<thead>
<tr>
<th>LOCAL MAXIMUM</th>
<th>LOCAL MINIMUM</th>
</tr>
</thead>
</table>

e. (2 points) Find \( f''(x) \).

f. (2 points) Find the partition numbers for \( f''(x) \).

g. (4 points) Make a sign chart for \( f''(x) \).

h. (2 points) Determine the interval(s) on which the graph of \( f(x) \) is concave downward? Give answer in interval notation.

i. (2 points) Give the x-coordinate of the inflection point(s) of \( f(x) \).
9. Below is a plot of the circle described by the equation 
\[ x^2 + y^2 = 1 \]

a. (4 points) Find an equation for \( \frac{dy}{dx} \) where \( y = y(x) \).

b. (2 points) Find the points on the circle where \( \frac{dy}{dx} = 0 \). Give the solution(s) as an ordered pair.

c. (2 points) On the plot given above, draw the tangent line to the graph at the points found in part (b) where \( \frac{dy}{dx} = 0 \).