Directions:
- Read and follow all directions carefully.
- You must show all work or justify your reasoning with meaningful evidence to receive full credit, regardless of whether it is explicitly asked for in the question.
- Clearly indicate your final answer for each problem.
- No use of calculators, books, notes, cell phones, or other electronic devices allowed during the exam.

1. (5 points each) Find the derivative $y'$ for each curve/function. Do not simplify.
   a. $y = x^2 e^{3x}$
   b. $y = \frac{x^3}{4x + 5}$
   c. $y = (6x + \ln x)^{-10}$
2. (5 points each) Evaluate each limit. When L'Hôpital's Rule is applied, justify why. Answer with $\infty$ or $-\infty$ where appropriate.

   a. 
   \[
   \lim_{x \to \infty} \frac{x^2}{2e^x}
   \]

   b. 
   \[
   \lim_{x \to 1} \frac{x^9 - x}{x^4 - 2x^3 + 1}
   \]

3. (8 points) Use implicit differentiation to find $y' = \frac{dy}{dx}$ for the following equation.

   \[x^2y + e^x + y^3 = 5\]
4. (10 points) Do ONE of the following TWO problems. Clearly indicate your chosen problem and box your final solution. If you attempt both, the better effort will be counted, but no extra credit will be given for the other problem. Include units in your answer.

a. An office supply company sells $x$ custom ball-point pens per year at $p$ per pen. The price-demand equation for the pens is $p = 6 - \sqrt{x}$. This means that the revenue function is $R = p \cdot x = (6 - \sqrt{x})x$. What price ($p$) should the company charge for the pens, and how many pens should be produced ($x$) to realize the maximum revenue?

b. A homeowner wants to fence off a two-section rectangular dog kennel against his house, as shown in the diagram. No fencing is to be used against the house. The homeowner has 80 feet of fence. What should the dimensions ($x$ and $y$) be in order to maximize the total enclosed area?

Not on Exam 2
5. This question deals with the following related rates problem:
Oil leaking from a tanker forms a circular slick. Right now (at the present moment), the area of the slick is 10 $mi^2$ and is increasing at a rate of 2 $mi^2$ per hour.

\[
Area = A = \pi r^2
\]

a. (5 points) Find the radius of the oil slick (in miles) when the area of the slick is 10 $mi^2$.

b. (6 points) How fast is the radius of the oil slick growing (right now) in miles per hour?

6. (8 points) Find the absolute maximum and minimum values of the function
\[f(x) = x^3 - 6x^2 + 9x + 2\] on the interval [1, 5].

\[
\begin{array}{|c|c|}
\hline
\text{Absolute Maximum} & f(\underline{\text{____}}) = \underline{\text{____}} \\
\hline
\text{Absolute Minimum} & f(\underline{\text{____}}) = \underline{\text{____}} \\
\hline
\end{array}
\]

\[\text{Not on Exam 2}\]
7. For the function \( y = f(x) = x^2 - 8 \ln x \), do the following:

a. (2 points) State the domain of \( f(x) \).

b. (6 points) Find \( f'(x) \), and show that it can be written as \( \frac{2(x-2)(x+2)}{x} \).

c. (10 points) Find the intervals of increase and decrease and all local extrema for the graph of \( f(x) \). Fill in your answers in the table provided. Answer NONE where appropriate.

<table>
<thead>
<tr>
<th>Interval(s) where ( f(x) ) is increasing:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Interval(s) where ( f(x) ) is decreasing:</td>
<td></td>
</tr>
<tr>
<td>( x ) – value(s) where local maxima occur:</td>
<td></td>
</tr>
<tr>
<td>( x ) – value(s) where local minima occur:</td>
<td></td>
</tr>
</tbody>
</table>
9. (10 points) Given the following information, sketch the graph of $f(x)$.

Domain: All real numbers except $x = 1$ and $x = -4$

$\lim_{x \to 1^+} f(x) = \infty$ and $\lim_{x \to 1^-} f(x) = -\infty$

$\lim_{x \to -4^+} f(x) = -\infty$ and $\lim_{x \to -4^-} f(x) = \infty$

$\lim_{x \to \infty} f(x) = 2$ and $\lim_{x \to -\infty} f(x) = 2$

<table>
<thead>
<tr>
<th>$x$</th>
<th>-6</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x)$</td>
<td>4</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>4</td>
</tr>
</tbody>
</table>

$f'(x)$

$\text{ND}$ $0$ $\text{ND}$

$\text{+++++++} \quad \text{++++++++++++} \quad \text{-----------} \quad \text{-----------}$

$\text{+++++++} \quad \text{-----------} \quad \text{-----------} \quad \text{+++++++}$

$\lim_{x \to -\infty} f(x) = 2$ and $\lim_{x \to \infty} f(x) = 2$