MATH 220 - Lecture 24 (11/07/2013)

Computer project - due Thursday, Dec 5.

One of the main goals of the computer project is to make yourselves familiar with MATLAB. You have access to MATLAB through the web portal at http://my.math.wsu.edu .

We present the session (equivalent ☺!) in MATLAB seen during the lecture to the right.

```
%% commands from the MATLAB session in Lecture 24 on Thursday,
%% November 7, 2013.

%% To illustrate row reduction, we used Problem 5 from Page 157 of the
%% book, which was solved in Lecture 23. In this problem, 3-vectors
%% b1, b2, and x are given, and we are asked to the B-coordinates of x
%% in the basis B = {b1,b2}.

%% You should check out some of the MATLAB tutorials listed in the
%% Computer Project description.

%% We can use the % sign to add comments - MATLAB ignores anything
%% written in a line following a % sign. Extra comments are added in
%% between the MATLAB commands here to illustrate.


>> b1 = [1
 4
-3]
b1 =
     1
     4
    -3

% A ' (prime) transposes a matrix or a vector when added to its
% end. Also, if you do not want MATLAB to display the output of a
% command, end the same with a ; (semi-colon).

>> b2 = [-2 -7 5]';

>> x = [2 9 -7]';
>> AugMtx = [b1 b2 x]
AugMtx =
     1    -2     2
     4    -7     9
    -3     5    -7

% Error messages in MATLAB - usually point out where the source of
% error is. Or, at least tell you from where things go wrong.

>> AugMtx = [b1 b2 x']
??? Error using ==> horzcat CAT
arguments dimensions are not consistent.

% In the above command, x' is 1 x 3, while b1 and b2 are both
% 3 x 1. Hence the dimensions do not match.

>> AugMtx = [b1 b2 x];

>> rref(AugMtx)
ans =
     1     0     4
     0     1     1
     0     0     0
```

You need to be aware of at least the basic commands related to matrix/vector operations in MATLAB.

```
>> help rref
 RREF   Reduced row echelon form.
    R = RREF(A) produces the reduced row echelon form of A.

    [R,jb] = RREF(A) also returns a vector, jb, so that:
       r = length(jb) is this algorithm's idea of the rank of A,
       x(jb) are the bound variables in a linear system, Ax = b,
       A(:,jb) is a basis for the range of A,
       R(1:r,jb) is the r-by-r identity matrix.

    [R,jb] = RREF(A,TOL) uses the given tolerance in the rank tests.

    Roundoff errors may cause this algorithm to compute a different
    value for the rank than RANK, ORTH and NULL.

    Class support for input A:
      float: double, single

    See also RANK, ORTH, NULL, QR, SVD.

% You could ask MATLAB to print rational numbers rather than decimal
% numbers. Here is an example.

>> rref([b1 b2 x/3])
ans =
    1.0000         0    1.3333
         0    1.0000    0.3333
         0         0         0

>> format rat

>> rref([b1 b2 x/3])
ans =
       1            0            4/3
       0            1            1/3
       0            0            0


% Following are the two determinant calculations we did class by
% hand. As illustrated here, you could verify any determinant
% calculations you are doing in the homework using MATLAB!

>> det([0 5 1; 4 -3 0; 2 4 1])
ans =
       2
>> det([1 -2 5 2; 0 0 3 0; 2 -6 -7 5; 5 0 4 4])
ans =
      -6
```

Several computations have functions in built (or, implemented) already in MATLAB. In particular, rref (reduced row echelon form), det (determinant), rank (rank), inv (inverse), are quite useful.

We will revisit the actual problems described in the project once we introduce eigenvalues and eigenvectors.

Determinant of $A \in \mathbb{R}^{n \times n}$ by expanding along Row-1

As illustrated in the previous lecture, we could compute the determinant of any square matrix by expanding along its Row-1.

In general, for $A \in \mathbb{R}^{n \times n}$ with

$$A = \begin{bmatrix} \boxed{a_{11} \quad a_{12} \quad \cdots \quad a_{1n}} \\ a_{21} \quad a_{22} \quad \cdots \quad a_{2n} \\ \vdots \qquad \ddots \qquad \vdots \\ a_{n1} \quad a_{n2} \quad \cdots \quad a_{nn} \end{bmatrix}$$

$$\det A = a_{11} \det A_{11} - a_{12} \det A_{12} + a_{13} \det A_{13} - \cdots + (-1)^{n+1} a_{1n} \det A_{1n},$$

where A_{1j} is the $(n-1)\times(n-1)$ matrix obtained by removing Row 1 and Column j of A.

Pg 167, Prob 2. Compute the determinant by expanding along Row 1.

determinant

$$\textbf{2.} \quad \begin{vmatrix} 0 & 5 & 1 \\ 4 & -3 & 0 \\ 2 & 4 & 1 \end{vmatrix}$$

$[\ ]$ ← notation for a matrix

$|\ |$ ← determinant of the matrix written inside

$$= 0 \cdot \begin{vmatrix} -3 & 0 \\ 4 & 1 \end{vmatrix} - 5 \begin{vmatrix} 4 & 0 \\ 2 & 1 \end{vmatrix} + 1 \begin{vmatrix} 4 & -3 \\ 2 & 4 \end{vmatrix}$$

$$= 0(-3\times1 - 4\times0) - 5(4\times1 - 2\times0) + 1(4\times4 - 2\times{-3})$$

$$= 0 - 20 + 22 = 2.$$

In fact we can expand along **any row** or **any column** to evaluate the determinant.

The result is given as Theorem 1 in the book.

Define $\underset{\text{cofactor}}{C_{ij}} = (-1)^{i+j} \det A_{ij}$ → remove row i column j from matrix A

The (i,j)-th cofactor of a matrix is the determinant of the submatrix obtained by removing Row i and Column j from the original matrix, multiplied by the appropriate sign that depends on $i+j$, i.e., by $(-1)^{i+j}$.

Expanding along column j:

$$\det A = a_{1j}C_{1j} + a_{2j}C_{2j} + \cdots + a_{nj}C_{nj}.$$

Expanding along Row i:

$$\det A = a_{i1}C_{i1} + a_{i2}C_{i2} + \cdots + a_{in}C_{in}.$$

Notice that the alternating $\pm$ signs are included in the cofactor values.

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Compute the determinants in Exercises 9–14 by cofactor expansions. At each step, choose a row or column that involves the least amount of computation.

10. $$\begin{array}{cc} & \begin{array}{cccc} & & 3 & \end{array} \\ \begin{array}{c} \\ 2 \\ \\ \\ \end{array} & \begin{vmatrix} 1 & -2 & 5 & 2 \\ \boxed{0 \quad 0 \quad 3 \quad 0} \\ 2 & -6 & -7 & 5 \\ 5 & 0 & 4 & 4 \end{vmatrix} \end{array}$$

We look for a row or a column with lots of zeros, and expand along that row/column. We repeat this idea for the 3x3 determinant in the next step.

$$= 3\cdot(-1)^{(2+3)} \begin{vmatrix} 1 & \boxed{-2} & 2 \\ 2 & \boxed{-6} & 5 \\ 5 & \boxed{0} & 4 \end{vmatrix} =$$

$$= -3\left((-2)\cdot(-1)^{(1+2)} \begin{vmatrix} 2 & 5 \\ 5 & 4 \end{vmatrix} + (-6)\cdot(-1)^{(2+2)} \begin{vmatrix} 1 & 2 \\ 5 & 4 \end{vmatrix}\right)$$

$$= -3\left(2\,(8-25) + (-6)\,(4-10)\right)$$

$$= -3\,(-34+36) = -6.$$