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Math 201 - Test 2

Section: _____ ***Group:*** _____ ***Name:*** Answers

Circle the correct answer on problems 1 and 9. Work will not be graded on these problems. On all other problems, show appropriate work to receive credit and place your answer in the blank provided. Point value of each problem is listed in the problem statement. ***Calculators are not allowed on this exam.***

1. Find the slope of the line connecting points with coordinates (–2, 4) and (–3, 1). *Circle the correct answer.* (4 points)

   a.) $\frac{1}{3}$

   b.) –3

   c.) $-\frac{1}{3}$

   (d.)) 3

   e.) $-\frac{3}{5}$

$$m = \frac{4-1}{-2-(-3)} = \frac{3}{1}$$

2. Find the equation in slope-intercept form of a line passing through the point with coordinates (–1, 2) and parallel to the line with equation $y = 2x - 4$. Sketch the line represented by your new equation. (6 points)

$m = 2$

$y - 2 = (x - (-1))(2)$

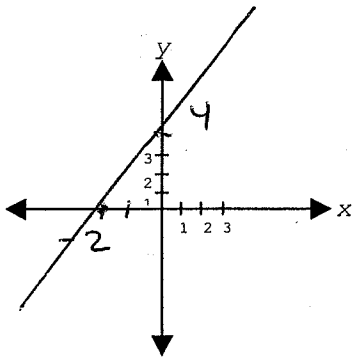
$= 2(x+1)$

$y - 2 = 2x + 2$

$y = 2x + 4$

Equation: $y = 2x + 4$

x	y
0	4
-2	0



3. Graph the quadratic function below. Identify the coordinates of the vertex, the x and y intercepts, state the range of the function and sketch. (8 points)

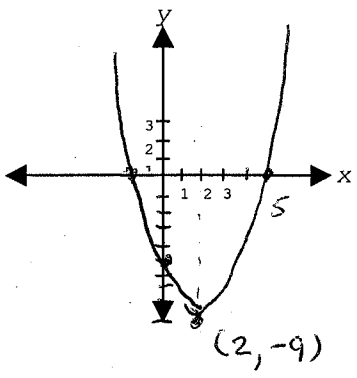
$y = f(x) = x^2 - 4x - 5 = (x-5)(x+1)$

Coordinates of vertex: (2, -9)

x intercept(s): 5 , -1

y intercept(s): -5

Range: $[-9, \infty)$

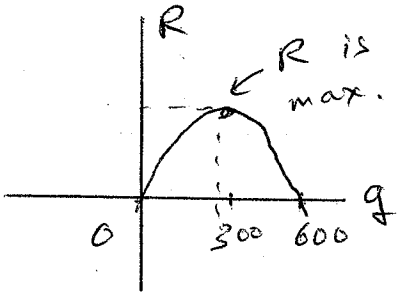


4. The revenue function for a manufacturer's product is $R = 1800q - 3q^2$, where R is the total revenue (in dollars) when q units are demanded (per week). Find the q value that maximizes the manufacturer's total revenue for a given week. (6 points)

$1800q - 3q^2$

$= 3q(600 - q)$

Vertex at $q = 300$



$q = 300$

5. Solve the system of equations below. *Show appropriate work.* (6 points)

$x = 4,\ y = 3$

① $2x - y = 5$

② $3x + 2y = 18$

① ×2: $4x - 2y = 10$

$+$

$7x = 28$

$x = 4$

$2(4) - y = 5$ $\quad y = 8 - 5 = 3$

6. Find the solution of the system of equations below algebraically. (6 points)

$\begin{cases} p = \sqrt{q} \\ p = q^2 \end{cases}$ $\quad q = p^2$

$p = (p^2)^2 = p^4$

$p^4 - p = 0$

$p(p^3 - 1) = 0$

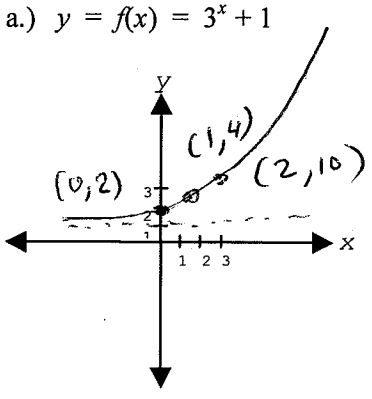
$p = 0,\ p = 1$

$q = 0 \quad q = 1$

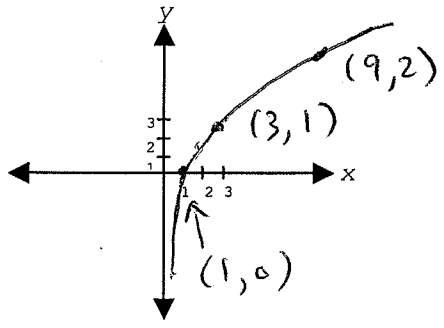
Solution: $\begin{matrix} p=0 \\ q=0 \end{matrix}\Big\}$ or $\begin{matrix} p=1 \\ q=1 \end{matrix}\Big\}$

7. Graph each function below on the axes provided. Place at least 3 labeled points on each graph. (5 points each)

a.) $y = f(x) = 3^x + 1$



b.) $y = f(x) = \log_3 x$.



Not drawn to scale.

8. Classify each of the following as True or False by circling the appropriate letter. **If the statement is false, justify your answer or provide a counter-example.** (3 points each)

T (F) a.) $\log_b 1 = 1$ $\log_{10} 1 = 0$ because $10^0 = 1$

(T) F b.) $\frac{1}{2}\log x = \log \sqrt{x}$ $\log x^r = r \log x$

$\log \sqrt{x} = \log x^{1/2} = \frac{1}{2}\log x$

T (F) c.) $\log_b(m+n) = \log_b m + \log_b n$

$\log_{10}(3+7) = \log_{10} 10 = 1 \neq \log_{10} 3 + \log_{10} 7$

T (F) d.) $\ln x - \ln y = \frac{\ln x}{\ln y}$ Take $x = y = e$

$\ln e - \ln e = 1 - 1 = 0$

but $\frac{\ln e}{\ln e} = \frac{1}{1} = 1$

9. Evaluate $2^{\log_2(x+1)}$. *Circle the correct answer.* (5 points)

a.) 2^{x+1}

(b.) $x+1$ $2^{\log_2(x+1)} = x+1$

c.) $4(x+1)$

d.) $(x+1)^2$

e.) None of the above

10. Write as a single logarithm with a coefficient of 1. (4 points) $3\ln x + 4\ln y - \ln z$ $\ln\left(\frac{x^3y^4}{z}\right)$

$3\ln x + 4\ln y - \ln z$

$= \ln x^3 + \ln y^4 - \ln z$

$= \ln \frac{x^3 y^4}{z}$

11. Write the expression in terms of $\ln x$, $\ln(x+1)$ and/or $\ln(x-2)$. (4 points)

$\ln \frac{x^3}{(x+1)(x-2)}$ $3\ln x - \ln(x+1) - \ln(x-2)$

$= \ln x^3 - \ln(x+1) - \ln(x-2)$

$= 3\ln x - \ln(x+1) - \ln(x-2)$

12. Solve each equation below. *Show appropriate steps.* (a and b: 4 points each; d-e: 5 points each)

a.) $3^{2x} = 81$

$x = 2$

$2x \log_3(3) = \log_3 81$

$2x = 4 \quad x = 2$

b.) $\log_2 32 = y$

$y = 5$

$32 = 2^y$

$y = 5$

c.) $\log_2(x+5) = 3 + \log_2(x-3)$

$x = 29/7$

$\log_2(x+5) - \log_2(x-3) = 3$

$\log_2 \frac{(x+5)}{(x-3)} = 3$

$\frac{x+5}{x-3} = 2^3 = 8$

$x+5 = 8(x-3)$

$x+5 = 8x-24$

$7x = 29$

$x = \frac{29}{7}$

d.) $\log_x(x+6) = 2$

$x = 3$

$x+6 = x^2$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 3$ or $x = -2$, $x = -2$ is not a solution (negative bases are not allowed)

e.) If $\ln 2 = a$, $\ln 9 = b$ and $\ln 7 = c$, solve the equation below and state your answer in terms of a, b and c.

$9(2^x) = 7$

$x = \frac{c-b}{a}$

$\ln 9 + x \ln 2 = \ln 7$

$b + ax = c$

$ax = c - b \qquad x = \frac{(c-b)}{a}$

13. The equation $A = P(1.075)^t$ gives the value A at the end of t years of an investment of P dollars compounded annually at an annual interest rate of 7.5%. How many years will it take for an investment to triple? *You may leave your answer in terms of logarithms.* (6 points)

$t = \ln 3 / \ln 1.075$

$3P = P(1.075)^t$

$3 = 1.075^t$

$t \ln(1.075) = \ln 3$

$t = \ln 3 / \ln 1.075$