Finite Mathematics (Fall 2008) – Solutions to Quiz 1

Version 1

1. (5) Solve the following equation: $\dfrac{1}{x-3} - \dfrac{3}{x-2} = \dfrac{4}{1-2x}$.

$$
\begin{aligned}
(1-2x)[(x-2)-3(x-3)] &= 4(x-3)(x-2) \\
(1-2x)(-2x+7) &= 4(x^2-5x+6) \\
(2x-1)(2x-7) &= 4x^2-20x+24 \\
4x^2-16x+7 &= 4x^2-20x+24, \text{which gives} \\
4x &= 17, \text{i.e.,} \\
x &= 17/4.
\end{aligned}
$$

None of the three denominators in the original equation are zero for this value of x. Hence, the solution is $x = 17/4$.

2. (5) Solve the following equation: $\sqrt{y-2}+2 = \sqrt{2y+3}$.

One clever way to solve this problem is to use the substitution $\sqrt{y-2} = u$. With this substitution in mind, we first write the original equation as follows:

$$
\begin{aligned}
\sqrt{y-2}+2 &= \sqrt{2y-4+4+3} = \sqrt{2(y-2)+7}. \quad \text{Using } u, \\
u+2 &= \sqrt{2u^2+7}. \quad \text{Squaring both sides,} \\
u^2+4u+4 &= 2u^2+7 \\
-u^2+4u-3 &= 0 \\
u^2-4u+3 = (u-1)(u-3) &= 0, \text{ giving } u = 1,3. \\
\text{Hence, } \sqrt{y-2} &= 1,3, \quad \text{giving } y-2 = 1^2, 3^2, \text{ or } 1,9, \\
\text{thus } y &= 3, 11.
\end{aligned}
$$

$u = \sqrt{y-2}$ is valid for both the values of y, and hence the solution set is $\{3,\ 11\}$.

Version 2

1. (5) Solve the following equation: $\dfrac{1}{x-2} - \dfrac{4}{x-3} = \dfrac{6}{1-2x}$.

$$\begin{aligned}
(1-2x)[(x-3)-4(x-2)] &= 6(x-2)(x-3) \\
(1-2x)(-3x+5) &= 6(x^2-5x+6) \\
(2x-1)(3x-5) &= 6x^2-30x+36 \\
6x^2-13x+5 &= 6x^2-30x+36, \text{which gives} \\
17x &= 31, \text{i.e.,} \\
x &= 31/17.
\end{aligned}$$

None of the three denominators in the original equation are zero for this value of x. Hence, the solution is $x = 31/17$.

2. (5) Solve the following equation: $\sqrt{y-2}+2 = \sqrt{2y-5}$.

One clever way to solve this problem is to use the substitution $\sqrt{y-2} = u$. With this substitution in mind, we first write the original equation as follows:

$$\begin{aligned}
\sqrt{y-2}+2 &= \sqrt{2y-4-1} = \sqrt{2(y-2)-1}. \quad \text{Using } u, \\
u+2 &= \sqrt{2u^2-1}. \quad \text{Squaring both sides,} \\
u^2+4u+4 &= 2u^2-1 \\
-u^2+4u+5 &= 0 \\
u^2-4u-5 = (u-5)(u+1) &= 0, \text{ giving } u = 5, -1. \\
\text{Hence, } \sqrt{y-2} &= 5, -1, \text{ giving } y-2 = 5^2, (-1)^2, \text{ or } 25, 1, \\
\text{thus } y &= 27, 3.
\end{aligned}$$

$u = \sqrt{y-2}$ is valid for both the values of y, and hence the solution set is $\{3, 27\}$.