B

Math 201 - Test 1

Section:________ **Name:** KEY

In problems 2 and 3 below indicate your choice by circling the correct solution. Work will not be graded. Work will not be graded on problem 14 as well. For all other problems, show appropriate work on the test page to receive credit and place your answer in the blank or space provided. Point value of each problem is given with the problem statement. ***Calculators are not allowed on this exam.*** *Formulas you may need on this exam:* $\frac{n(n+1)}{2}$, $\frac{n(n+1)(2n+1)}{6}$, $\frac{n^2(n+1)^2}{4}$

1. Solve for the variable in each equation below : (5 points each)

a.) $\frac{2t-1}{\cancel{8}} = \frac{\cancel{10}\,6t-3}{\cancel{5}} + \frac{t}{\cancel{10}}$

$5(2t-1) = 2(6t-3) + t$

$10t - 5 = 12t - 6 + t$

$6 - 5 = 3t \qquad t = \frac{1}{3}$

$\boxed{t = \frac{1}{3}}$

b.) $\frac{8}{x^2-1} = \frac{1}{x+1} - \frac{1}{x-1}$

$8 = (x-1) - (x+1) = -2$ No solution!

no solution

2. In the literal equation below, solve for t in terms of the remaining variables. (5 points) *(Circle the correct answer.)*

$mt + gt = 2(3 + t)$

A) $6 - m - g$ B) $\frac{6}{mg-2}$ C) $\frac{6}{mg+2}$ D) $\frac{6}{m+g+2}$ E) $\frac{6}{m+g-2}$

$t(m+g) = 6 + 2t$

$t(m+g-2) = 6$

$t = \frac{6}{m+g-2}$

3. The sum of the root(s), that is the solution(s), of the equation $\left(\sqrt{x+2}\right)^2 = (x-4)^2$ is: (5 points) *(Circle the correct answer.)*

A) 0 B) 9 C) – 5 D) 5 E) 7

$x + 2 = x^2 - 8x + 16$

$0 = x^2 - 8x - x + 16 - 2$

$0 = x^2 - 9x + 14$

$(x-7)(x-2) = 0 \quad x = 7, 2$

~~7+2=9~~

But $x = 2$ does not work!

$\sqrt{2+2} = 2 - 4 = -2$

$\sqrt{4} = -2$

$2 \neq -2$

$\boxed{\text{So, only root is } x = 7}$

4. Solve for x: $x^4 - 7x^2 + 12 = 0$ (5 points)

$\{2, -2, \sqrt{3}, -\sqrt{3}\}$

$u = x^2$: $u^2 - 7u + 12 = 0$

$(u-3)(u-4) = 0$, $u = 3, 4$

$x^2 = 3, 4$

So $x = \pm\sqrt{3}, \pm\sqrt{4}$ i.e, $\boxed{x = \pm\sqrt{3}, \pm 2}$

5. In each application problem below represent all unknowns in terms of one variable, establish an appropriate equation and solve. *Show all appropriate steps.* (7 points each)

a.) A homeowner needs to spray her trees to kill defoliating insects. She needs 128 ounces of a solution made up of 3 parts insecticide A and 5 parts of insecticide B. How many ounces of insecticide B should be used?

Let x = # ~~parts~~ ounces of insecticide B

Then # ounces of insecticide A = $\left(\frac{3}{5}\right)x$

Total ounces = $x + \left(\frac{3}{5}\right)x = 128$

$5x + 3x = 128 \times 5$; $x = \frac{\cancel{128}^{16} \times 5}{\cancel{8}} = 80$

$\boxed{\text{80 ounces of insecticide B is used.}}$

b.) A company's revenue from the sale of interior doors is \$98 per door. The cost of producing the doors is \$48 per door with \$14,000 in fixed costs. How many doors must the company sell in order to break even?

Let x = # doors

Total cost = fixed cost + variable cost

= $14000 + 48x$

Total revenue = $98x$.

for Break even: revenue = cost

$98x = 48x + 14000$

$50x = 14000$

$x = \frac{1400\cancel{0}}{5\cancel{0}} = 280$

$\boxed{\text{280 Doors must be made}}$

6. Solve for x in each inequality below. State your solutions using interval notation. (5 points each)

a.) $\frac{6x-2}{3} \geq -\frac{1}{4}$

$6x - 2 \geq -\frac{3}{4}$

$6x \geq 2 - \frac{3}{4} = \frac{5}{4}$

$x \geq \frac{5}{4 \times 6} = \frac{5}{24}$

$\left[\frac{5}{24}, \infty\right)$

b.) $\left|\frac{2x+8}{3}\right| \geq 4$

$\frac{2x+8}{3} \geq 4$ | $\frac{2x+8}{3} \leq -4$

$2x + 8 \geq 12$ | $2x + 8 \leq -12$

$2x \geq 4$ | $2x \leq -20$

$x \geq 2$ | $x \leq -10$

$(-\infty, -10] \cup [2, \infty)$

7. Evaluate $\sum_{i=1}^{20} 14i$. (3 points) $\boxed{2940}$

$$= 14 \sum_{i=1}^{20} i = 14\left(\frac{20(20+1)}{2}\right) = \frac{14 \cdot \cancel{20}^{10} \cdot 21}{\cancel{2}} = 2940$$

8. State the degree and leading coefficient of the polynomial function $f(x) = -3x^8 + 2x^5 + 4$. (1 point each)

Leading Coefficient: -3

Degree: 8

9. Given the function $g(x) = \dfrac{x}{x-3}$, find the domain of function g, $g(0)$, $g(1/2)$ and $g(x^2)$. Simplify your answer if needed. (2 points each)

$$g(\tfrac{1}{2}) = \frac{\frac{1}{2}}{\frac{1}{2}-3} = \frac{\frac{1}{2}}{\frac{1}{2}-\frac{6}{2}} = \frac{1}{1-6} = \frac{-1}{5}$$

$$g(x^2) = \frac{x^2}{x^2-3}$$

Domain: All reals except 3

$g(0) =$ 0

$g(1/2) =$ $-\frac{1}{5}$

$g(x^2) =$ $\frac{x^2}{x^2-3}$

10. If $f(x) = x^2 - 1$ and $g(x) = x + 2$, find: (3 points each)

a.) $(f+g)(x)$ $= f(x)+g(x) = x^2-1+x+2 = x^2+x+1$ $\qquad$ x^2+x+1

b.) $(f \times g)(2)$ $= f(2)\cdot g(2) = ((2)^2-1)\cdot(2+2)$
$= (4-1)\cdot(4) = 3\cdot 4 = 12$ $\qquad$ 12

c.) $f(g(x))$ $= [g(x)]^2-1 = (x+2)^2-1 = x^2+4x+4-1$ $\qquad$ x^2+4x+3

d.) $(g \circ f)(x)$ $= g(f(x))$
$= f(x)+2 = x^2-1+2 = x^2+1$ $\qquad$ x^2+1

11. Find the inverse of the function below if it exists, *showing all appropriate steps*. If the inverse doesn't exist, so state. (5 points)

$f(x) = 3x - 5$

Let $g(x)$ be the inverse. Then
$f(g(x)) = x$.
i.e., $3(g(x)) - 5 = x$
Solving for $g(x)$, $3g(x) = x+5$
so, $g(x) = \frac{1}{3}x + \frac{5}{3}$

$\boxed{f^{-1}(x) = \frac{1}{3}x + \frac{5}{3}}$

12. Given the equation: $y = 4 - 2x^2$: (8 points)

a.) Sketch the graph on the axes at right.

b.) Identify the intercepts $y=0: \; 4-2x^2=0, \; x^2=2, \; x=\pm\sqrt{2}$

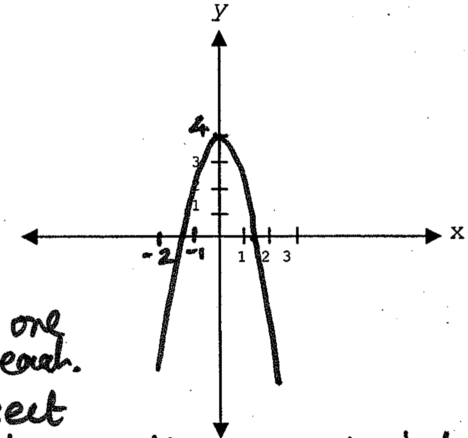
- x intercept: $(\sqrt{2},0), (-\sqrt{2},0)$
- y intercept: $(0,4)$ $x=0: \; y=4$

c.) Based on your graph, is y a function of x? yes. Vertical lines intersect graph at one point each.

d.) If y is a function of x, is it one-to-one? No. Horizontal lines intersect graph at more than one point.

e.) If y is a function of x, what are the domain and range?

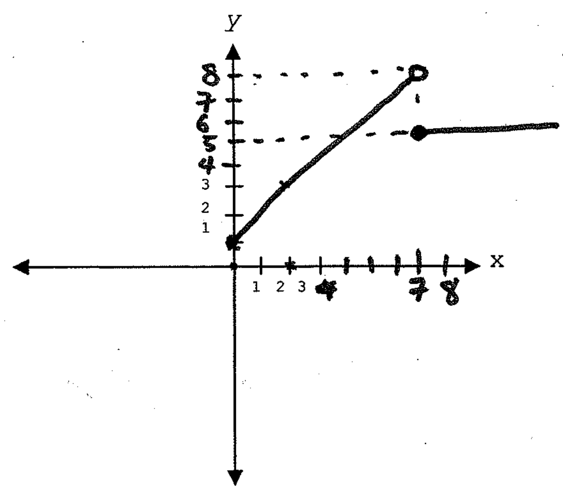
- Domain: all real values
- Range: all real values ≤ 4 $(-\infty, 4]$



13. Graph the case defined function below on the axes provided. (4 points)

$$y = f(x) = \begin{cases} x+1 & \text{if } 0 \leq x < 7 \\ 5 & \text{if } x \geq 7 \end{cases}$$

x	y
0	1
2	3
8	5



14. Complete the table below indicating the types of symmetry of the graph of each equation. The first row is completed to illustrate an appropriate response. (1 point each cell)

Equation	*x-axis symmetry*	*y-axis symmetry*	*origin symmetry*
$y = 5x$	NO	NO	YES
$y = x^2 - 4$ $y=(-x)^2-4 = x^2-4$	NO $-y = x^2-4$	YES	NO $y = -x^2+4$
$x^2 + xy + y^2 = 0$	NO $x^2 - xy + (-y)^2 = 0$	NO $(-x)^2 - xy + y^2 = 0$	YES $(-x)^2 + xy + (-y)^2 = 0$
$y = \sqrt{x^2 - 25}$	NO $-y = \sqrt{x^2-25}$ $y = -\sqrt{x^2-25}$	YES $y = \sqrt{(-x)^2-25} = \sqrt{x^2-25}$	NO $-y = \sqrt{(-x)^2-25}$ $y = -\sqrt{x^2-25}$