

Exam 2

Financial Mathematics

Exam 2 is a three-hour, multiple-choice examination that is administered by Preliminary Actuarial Examinations/SOA and is identical to SOA Exam FM. The examination is jointly sponsored and administered by the CAS, SOA, and the Canadian Institute of Actuaries (CIA). The examination is also jointly sponsored by the American Academy of Actuaries (AAA) and the Conference of Consulting Actuaries (CCA).

Exam 2 is administered as a computer-based test. For additional details, please refer to “Computer-Based Testing Rules and Procedures” (<http://www.beanactuary.org/exams/cbt.cfm>).

The goal of the syllabus for this examination is to provide an understanding of the fundamental concepts of financial mathematics, and how those concepts are applied in calculating present and accumulated values for various streams of cash flows as a basis for future use in: reserving, valuation, pricing, asset/liability management, investment income, capital budgeting, and valuing contingent cash flows. The candidate will also be given an introduction to financial instruments, including derivatives, and the concept of no-arbitrage as it relates to financial mathematics.

Exam 2 assumes a basic knowledge of calculus and an introductory knowledge of probability.

The following learning objectives are presented with the understanding that candidates are allowed to use specified calculators on the exam. The education and examination of candidates reflects that fact. In particular, such calculators eliminate the need for candidates to learn and be examined on certain mathematical methods of approximation.

Please check the “Syllabus Updates” section of the CAS Web Site for any changes to the exam or syllabus. Information about Study Notes is available in the “Study Resources” section.

LEARNING OBJECTIVES

I. Interest Theory

A. Time Value of Money

1. The candidate will be able to define and recognize the definitions of the following terms:
 - a. Interest rate (rate of interest)
 - b. Simple interest
 - c. Compound interest
 - d. Accumulation function
 - e. Future value
 - f. Present value/net present value
 - g. Discount factor
 - h. Discount rate (rate of discount)
 - i. Convertible m -thly
 - j. Nominal rate
 - k. Effective rate
 - l. Force of interest
 - m. Equation of value
2. The candidate will be able to:
 - a. Given any two of interest rate, present value, or future value, calculate the third based on simple or compound interest.
 - b. Given any one of the effective interest rate, the nominal interest rate convertible m -thly, the effective discount rate, the nominal discount rate convertible m -thly, or the force of interest, calculate all of the other items.

- c. Write the equation of value given a set of cash flows and an interest rate.

B. Annuities with payments that are not contingent

1. The candidate will be able to define and recognize the definitions of the following terms:
 - a. Annuity-immediate
 - b. Annuity-due
 - c. Perpetuity
 - d. Payable m-thly
 - e. Level payment annuity
 - f. Arithmetic increasing/decreasing payment annuity
 - g. Geometric increasing/decreasing payment annuity
 - h. Term of annuity
2. The candidate will be able to:
 - a. Given an annuity with level payments, immediate (or due), payable m-thly, and any three of present value, future value, interest rate, payment, and term calculate the remaining two items.
 - b. Given an annuity with non-level payments, immediate (or due), payable m-thly, the pattern of payment amounts, and any three of present value, future value, interest rate, payment amounts, and term of annuity calculate the remaining two items.

C. Loans

1. The candidate will be able to define and recognize the definitions of the following terms:
 - a. Principal
 - b. Interest
 - c. Term of loan
 - d. Outstanding balance
 - e. Final payment (drop payment, balloon payment)
 - f. Amortization
 - g. Sinking fund
2. The candidate will be able to:
 - a. Given any four of term of loan, interest rate, payment amount, payment period, principal, calculate the remaining items.
 - b. Calculate the outstanding balance at any point in time.
 - c. Calculate the amount of interest and principal repayment in a given payment.
 - d. Given the quantities, except one, in a sinking fund arrangement calculate the missing quantity.

D. Bonds

1. The candidate will be able to define and recognize the definitions of the following terms:
 - a. Price
 - b. Redemption value
 - c. Par Value/Face value
 - d. Coupon, Coupon rate
 - e. Term of bond
 - f. Yield rate
 - g. Callable/non-callable
 - h. Book value
 - i. Accumulation of discount
2. The candidate will be able to:
 - a. Given any four of price, redemption value, yield rate, coupon rate, and term of bond, calculate the remaining item.

E. General Cash Flows and Portfolios

1. The candidate will be able to define and recognize the definitions of the following terms:
 - a. Yield rate/rate of return
 - b. Dollar-weighted rate of return/Time-weighted rate of return
 - c. Current value
 - d. Duration (Macaulay and modified)
 - e. Convexity
 - f. Portfolio
 - g. Spot rate
 - h. Forward rate
 - i. Yield curve
 - j. Stock price, stock dividend
2. The candidate will be able to:
 - a. Calculate the current value of a set of cash flows.
 - b. Calculate the portfolio yield rate.
 - c. Calculate the dollar-weighted and time-weighted rate of return.
 - d. Calculate the duration and convexity of a set of cash flows.
 - e. Calculate either Macaulay or modified duration given the other.
 - f. Use duration and convexity to approximate the change in present value due to a change in interest rate.
 - g. Calculate the price of a stock using the dividend discount model.

F. Immunization

1. The candidate will be able to define and recognize the definitions of the following terms:
 - a. Cash-flow matching;
 - b. Immunization (including full immunization);
 - c. Redington immunization.
2. The candidate will be able to:
 - a. Construct an investment portfolio to fully immunize a set of liability cash flows.
 - b. Construct an investment portfolio to match present value and duration of a set of liability cash flows.
 - c. Construct an investment portfolio to exactly match a set of liability cash flows.

II. Financial Economics

A. General Derivatives

1. The candidate will be able to define and recognize the definitions of the following terms:
 - a. Derivative, Underlying asset, Over-the-counter market
 - b. Ask price, Bid price, Bid-ask spread
 - c. Short selling, Short position, Long position
 - d. Stock index
 - e. Spot price
 - f. Net profit/payoff
 - g. Credit risk
 - h. Marking-to-market
 - i. Margin, Maintenance margin, Margin call
2. The candidate will be able to evaluate an investor's margin position based on changes in asset values.

B. Options

1. The candidate will be able to define and recognize the definitions of the following terms:

- a. Call option, Put option
- b. Expiration, Expiration date
- c. Strike price/Exercise price
- d. European option, American option, Bermudan option
- e. In-the-money, At-the-money, Out-of-the-money
- f. Covered call, Naked writing
- g. Dividends
- h. Put-call parity

2. The candidate will be able to evaluate the payoff and profit of basic derivative contracts.

C. Hedging and Investment Strategies

1. The candidate will be able to define and recognize the definitions of the following terms:

- a. Hedging, Arbitrage
- b. Diversifiable risk, Nondiversifiable risk
- c. Synthetic forwards
- d. Spreads (including bull, bear, box, and ratio spreads)
- e. Collars (including zero-cost collars), Paylater strategy
- f. Straddles (including strangles, written straddles and butterfly spreads)
- g. Convertible bond, Mandatorily convertible bond

2. The candidate will be able to:

- a. Explain how derivative securities can be used as tools to manage financial risk.
- b. Explain the reasons to hedge and not to hedge.
- c. Evaluate the payoff and profit of hedging strategies.

D. Forwards and Futures

1. The candidate will be able to define and recognize the definitions of the following terms:

- a. Forward contract, Prepaid forward contract
- b. Outright purchase, Fully leveraged purchase
- c. Implied repo rate
- d. Cost of carry
- e. Lease rate
- f. Futures contract

2. The candidate will be able to:

- a. Determine forward price from prepaid forward price.
- b. Explain the relationship between forward price and futures price.
- c. Explain the relationship between forward price and future stock price.
- d. Use the concept of no-arbitrage to determine the theoretical value of futures and forwards.
- e. Given any four of call premium, put premium, forward price, strike price and interest rate, calculate the remaining item using the put-call parity formula.

E. Swaps

1. The candidate will be able to define and recognize the definitions of the following terms:

- a. Swap, Prepaid swap
- b. Swap term, Swap spread, Notional Amount
- c. Simple commodity swap, Interest rate swap
- d. Deferred swap

2. The candidate will be able to use the concept of no-arbitrage to determine the theoretical values of swaps.

Text References for Exam 2

Knowledge and understanding of the financial mathematics concepts are significantly enhanced through working out problems based on those concepts. Thus, in preparing for the Financial Mathematics exam, whichever of the source textbooks candidates choose to use, candidates are encouraged to work out the textbook exercises related to the listed readings.

Suggested Textbooks for Learning Objectives in Section I, Interest Theory

There is not a single textbook required for the learning objectives in Section I. The texts listed below are representative of the textbooks available to cover the material on which the candidate may be tested. Not all topics may be covered at the same level in each text. The candidate may wish to use one or more texts in his/her preparation for the examination.

<p>Broverman, S.A., <i>Mathematics of Investment and Credit</i> (Fourth Edition), 2008, ACTEX Publications:</p> <ul style="list-style-type: none"> Chapter 1 (1.1-1.7) Chapter 2 (2.1 -2.4 excluding 2.4.2 and 2.4.3) Chapter 3 (3.1-3.3, excluding 3.2.1 and 3.2.2) Chapter 4 (4.1-4.3.1) Chapter 5 (5.1-5.3 excluding 5.1.4 and 5.3.2) Chapter 6 (6.1-6.3 excluding 6.2) Chapter 7 (7.1-7.2) Chapter 8 (8.1, 8.3.1 and 8.4.1–8.4.2) 	
<p>Daniel, J.W.; and Vaaler, L.J.F., <i>Mathematical Interest Theory</i> (Second Edition), 2009, The Mathematical Association of America:</p> <ul style="list-style-type: none"> Chapter 1 (1.3-1.12, 1.14) Chapter 2 (2.2-2.7) Chapter 3 (3.2-3.9, 3.11, 3.13) Chapter 4 (4.2-4.6) Chapter 5 (5.2-5.4) Chapter 6 (6.2-6.6, 6.9) Chapter 7 (7.1) Chapter 8 (8.3) Chapter 9 (9.1-9.5) <p>Note: Candidates may also use the First Edition of <i>Mathematical Interest Theory</i> (Publisher: Prentice Hall). The same chapter references apply.</p>	
<p>Kellison, S.G., <i>The Theory of Interest</i> (Third Edition), 2008, Irwin/McGraw-Hill:</p> <ul style="list-style-type: none"> Chapter 1 (1.2-1.10) Chapter 2 (2.3-2.6) Chapter 3 (3.2-3.8) Chapter 4 (4.2-4.9) Chapter 5 (5.2-5.6) Chapter 6 (6.2-6.7, 6.10) Chapter 7 (7.2-7.7) Chapter 9 (9.4) Chapter 10 (10.2-10.5) Chapter 11 (11.2-11.8) 	

<p>Ruckman, C.; and Francis, J., <i>Financial Mathematics: A Practical Guide for Actuaries and other Business Professionals</i> (Second Edition), 2005, BPP Professional Education:</p> <p>Chapter 1 Chapter 2 Chapter 3 (3.1-3.9) Chapter 4 (4.1-4.5) Chapter 5 Chapter 6 (6.1-6.3 excluding 6.1.6-6.1.7) Chapter 7 (7.1-7.9) Chapter 8 (8.1-8.3)</p>	
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Textbook for Learning Objectives in Section II, Financial Economics

<p>McDonald, R.L., <i>Derivatives Markets</i> (Second Edition), 2006, Addison Wesley:</p> <p>Chapter 1 (1.1-1.4) Chapter 2 (2.1-2.6 and Appendix 2.A) Chapter 3 (3.1-3.5) Chapter 4 (4.1-4.4) Chapter 5 (5.1-5.4 and Appendix 5.B) Chapter 8 (8.1-8.2).</p>	
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Study Notes

<i>Derivatives Markets</i> , Errata, 2006 Second Edition, by R. McDonald: http://www.kellogg.northwestern.edu/faculty/mcdonald/htm/typos2e.html	W
“Notation and terminology used for Exam FM/2”	W
All versions of Exam 2/FM that have been released since 2000 are posted online at http://www.soa.org/education/exam-req/syllabus-study-materials/edu-multiple-choice-exam.aspx and in “Past Exams” under “Study Tools” in the “Admissions/Exams” section of the CAS Web Site (www.casact.org).	W
“Sample Questions and Solutions for <i>Derivatives Markets</i> .”	W
Review of Calculator Functions for the Texas Instruments BA-35, SOA Study Note FM-22-05.	W
Review of Calculator Functions for the Texas Instruments BA II Plus, SOA Study Note FM-23-05.	W

Source Key

NEW Indicates new or updated material or modified citation.

W Represents material that is available at no charge under “Web Notes” in the “Study Tools” section of the CAS Web Site.

Publishers and Distributors

Contact information is furnished for those who wish to purchase the text references cited for Exam 2/FM. Publishers and distributors are independent and listed for the convenience of candidates; inclusion does not constitute endorsement by the CAS.

ACTEX Publications (Mad River Books), 107 Groppo Drive, Suite A, P.O. Box 974, Winsted, CT 06098; telephone: (800) 282-2839 or (860) 379-5470; fax: (860) 738-3152; e-mail: retail@actexmadriver.com; Web site: www.actexmadriver.com.

Actuarial Bookstore, P.O. Box 69, Greenland, NH 03840; telephone: (800) 582-9672 (U.S. only) or (603) 430-1252; fax: (603) 430-1258; Web site: www.actuarialbookstore.com.

Broverman, S.A.; *Mathematics of Investment and Credit* (Fourth Edition) 2008, ACTEX Publications, 140 Willow Street, Suite One, P.O. Box 974, Winsted, CT 06098; telephone: (800) 282-2839 or (860) 379-5470; fax: (860) 738-3152; e-mail: retail@actexmadriver.com.

Daniel, J.W.; and Vaaler, L.J.F., *Mathematical Interest Theory* (Second Edition), 2009, The Mathematical Association of America, P.O. Box 91112, Washington, DC 20090-1112; telephone: (800) 331-1622 or (301) 206-9789; Web site: www.maa.org.

Kellison, S.G., *The Theory of Interest* (Third Edition), 2008, Irwin/McGraw-Hill; P.O. Box 182604, Columbus, OH 43272; telephone: (877) 833-5524; Web site: www.mcgraw-hill.com.

McDonald, R.L., *Derivatives Markets* (Second Edition), 2006, Addison Wesley, imprint of Pearson Education, Inc., 200 Old Tappan Road, Old Tappan, NJ 07675; Web site: <http://www.aw-bc.com/catalog/>.

Ruckman, C.; and Francis, J., *Financial Mathematics: A Practical Guide for Actuaries and other Business Professionals* (Second Edition), 2005, BPP Professional Education, 245 Hopmeadow Street, Weatogue, CT 06089; telephone: (888) 277-6001; Web site: www.bpptraining.com.

Society of Actuaries, 475 N. Martingale Road, Suite 600, Schaumburg, IL 60173-2226; telephone: (847) 706-3500; fax: (847) 706-3599; Web site: www.soa.org.

SlideRule Books, P.O. Box 69, Greenland, NH 03840; telephone: (877) 407-5433 or (603) 373-6140; fax: (877) 417-5433 or (603) 430-1258; Web site: www.sliderulebooks.com.