Study Habits and Attainment in Undergraduate Mathematics: A Social Network Analysis

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In this article, we argue that although mathematics educators are concerned about social issues, minimal attention has been paid to student–student interactions outside the classroom. We discuss social network analysis as a methodology for studying such interactions in the context of an undergraduate course. We present results on the questions: Who studies with whom? What are students’ study habits, and are these systematically related to the habits of those with whom they interact? Do individual and collaborative study habits predict attainment? We discuss the implications of these findings for research on undergraduate learning and on social issues in mathematics education, suggesting that social network analysis may provide a bridge between mathematics education researchers who focus on cognitive and on social issues.

*Keywords*: Higher education; Social capital; Social network analysis; Study habits; Undergraduate mathematics

Many factors affect students’ engagement with and success in advanced mathematics, and these have been investigated using diverse perspectives and methodologies. Some research has considered individual cognitive or behavioral factors, often using a quantitative approach. For instance, students who are more conscientious engage more consistently with available learning resources (Alcock, Attridge, Kenny, & Inglis, 2014), and those who engage more consistently with lectures (Inglis, Palipana, Trenholm, & Ward, 2011) or with peer-assisted learning (Duah, Croft, & Inglis, 2014) perform better on assessments. Some important factors in learning, however, are not individual but social. Students do not study in isolation; their peers form an important part of the learning environment. And mathematics educators are concerned about the role of social interactions at all educational levels (Rasmussen & Marrongelle, 2006; Yackel & Cobb, 1996). There is a body of research on mathematical discourse (see Ryve, 2011, for a review), and researchers have discussed both the “social turn” in mathematics education (Goos, Galbraith, & Renshaw, 2002; Inglis & Foster, 2018; Lerman, 2000) and the sociopolitical environment in which learning occurs (Boaler, 2008; Gutiérrez, 2013; Jablonka, Wagner, & Walshaw, 2012). Some theorists are particularly concerned with the social capital to which students do
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or do not have access and the ways in which this affects their educational success (e.g., Cobb, Stephan, McClain, & Gravemeijer, 2011; Hernandez-Martinez & Williams, 2013).

Methodologically, empirical studies of social factors often report detailed qualitative analyses of short sequences of classroom interactions (Ryve, 2011). In undergraduate mathematics education, such studies have often been based in small specialist classes, and reports have focused on teacher and student activity (Johnson, Caughman, Fredericks, & Gibson, 2013), on guided reinvention task sequences (Larsen, 2013), or on mathematical arguments in small-group or whole-class settings (Larsen & Zandieh, 2008). But there is also growing interest in social interactions in a broader range of larger, less specialist classes and in using alternative methods. Both research and development projects have investigated inquiry-based learning (Hayward, Kogan, & Laursen, 2016), and large-scale survey research has shown that persistence in U.S. college mathematics is related to student perceptions of factors such as frequency of whole-class discussion and the extent to which explanation is expected in class (Ellis, Kelton, & Rasmussen, 2014). This work contributes to our understanding of ways to orchestrate interactions and of the effects of different pedagogies. But it tells us little about student approaches to studying outside the classroom and little about patterns of social interaction in the broader educational environment.

Fortunately, it is possible to study social interactions at a larger scale, and social network analysis (SNA) provides one way to do this (Carolan, 2013). The basic approach is simple: Participants are asked to list people with whom they interact around one or more issues and perhaps to state how often or with what intensity these interactions occur (Borgatti & Halgin, 2011). They might also be asked straightforward questions about their individual characteristics or habits. This information is used to build a social network model and to investigate questions at three levels: the whole cohort level, describing features such as the density of the network; the dyad level, using individual attributes to predict, for instance, who interacts with whom; and the individual level, using individual attributes plus the amount or quality of interaction to predict an outcome such as performance (Borgatti, Everett, & Johnson, 2013).

The SNA approach is arguably of particular relevance in higher education, where the greater requirement for independent study renders student–student interactions more important than they might be for younger pupils. This is especially true in countries that operate cohort models of higher education, because students who share classes throughout a degree program have many interaction opportunities. For instance, at the U.K. university in which the present study took place, mathematics students spend approximately 18 hours per week in lectures and tutorials; lectures can involve over 200 students, and the vast majority of students live on campus. This means that most students have little individual contact with lecturers and that collaborative independent study is easy to arrange. Students’ peers are therefore a natural resource for information about both practical matters and mathematical ideas. But at present we know very little about student–student interactions outside lectures or, indeed, about how mathematics students use their independent study time. In the present study, we documented independent study behaviors and used SNA to investigate interactions among
students in a large undergraduate mathematics course. Specifically, we addressed these questions:

1. Who studies with whom?
2. What are students’ study habits, and are these systematically related to the habits of those with whom they interact?
3. Do individual and collaborative study habits predict attainment?

**Theoretical Background**

SNA permits researchers to formulate and address questions using theoretical constructs that closely match those used in mathematics education. In this section, we consider social capital, homophily, and centrality, illustrating these using studies related to our own in discipline, structure, or both (social network analyses are rare in mathematics education but are becoming more common in broader educational research; see, e.g., McCormick, Fox, Carmichael, & Procter, 2010).

**Social Capital**

In SNA (Burt, 2000), in education (Dika & Singh, 2002), and in mathematics education (Choudry, Williams, & Black, 2017; Morgan & Sørensen, 1999), *social capital* is conceptualized in two distinct but related ways. Some researchers are interested in the collective social capital a unit holds by virtue of its internal relationships, where the theorized mechanism is that strong internal relationships enable the unit to perpetuate its ideals and interact effectively with external organizations (Bourdieu, 1986; Bridwell-Mitchell, 2017; Jorgensen, Gates, & Roper, 2014). Other researchers are interested in the social capital that individuals hold by virtue of their relationships, where the theorized mechanism is that social ties provide access to resources, whether physical, financial, or informational (Burt, 2000; Coleman, 1988; Goddard, 2003). In mathematics education, some researchers have used the concept of social capital to explain differences in student attainment or persistence. For example, Choudry, Williams, and Black (2017) found that small friendship networks mediate social capital and access to further cultural capital, which may explain why some students attain more than others. And Crosnoe and Schneider (2010) found that students with greater social capital persist in high school mathematics at higher rates than their disadvantaged peers, even when they have the same initial placements and skill levels.

The study reported here focuses on individual behavior and attainment within a large but bounded social group—an undergraduate lecture class. As such, its logic is that of social capital as accrued by individuals: Through connections to others, students might have more or less access to information useful for success. But information in the literal sense is only one thing that might pass through social ties. Ties also provide exposure to behaviors: Students whose friends study consistently might be more likely to do so too. Alternatively, social ties might arise as a consequence of preexisting behaviors: Students who study consistently might attract others who do the same (or might become useful resources for lazier peers). Documenting ties is therefore only the beginning of SNA. Understanding a social situation ideally involves understanding effects of ties on individual students and
effects of individual attributes on ties. These effects can be conceptualized in terms of homophily and diffusion.

**Homophily and Diffusion**

In a large range of studies, people have been found to be more similar to their *alters*—those with whom they interact—than they are to others (McPherson, Smith-Lovin, & Cook, 2001). This can be explained in three ways. *Homophily* (or *assortativity*) refers to the tendency people have to associate with others who have similar characteristics or habits. *Diffusion* (or *influence* or *contagion* or *induction*) refers to the process by which characteristics or habits spread through a population (Shalizi & Thomas, 2011). Finally, there might be *confounds*: Environmental factors might influence related actors similarly (Christakis & Fowler, 2007). Confounds, of course, are common to all correlational research and not specific to SNA.

Homophily and diffusion are sometimes considered by researchers and policymakers as phenomena to be leveraged or broken down. For instance, SNA has been used both to analyze online behavior for purposes of marketing products or spreading ideas (e.g., Centola, 2010) and to study the spread of undesirable health behaviors and outcomes such as smoking and obesity (e.g., Christakis & Fowler, 2013). In educational settings, interventions might seek to break down homophily to promote wider conversations among people with different experience. For instance, Coburn, Choi, and Mata (2010) collected data from elementary school teachers involved in a mathematics reform effort. They documented a desirable shift over time from small, homogenous, grade-level-focused networks to larger, more diverse networks providing better access to expertise.

Studies of homophily and diffusion can be methodologically complex because causality might be clear or ambiguous (Steglich, Snijders, & Pearson, 2010). For instance, people tend to have more friendships with those of the same gender (McPherson et al., 2001), and it is natural to infer that this is due to homophily. But some attributes are more readily changeable: Obese people might gravitate to one another, or obesity might diffuse across existing social groups because of shared habits or environmental influences (Christakis & Fowler, 2013). In this report, we investigate questions for which causality is clear and questions for which it is ambiguous. Causality is relatively clear when we examine the relationship between degree program and who discusses mathematics with whom: In the United Kingdom, degree program is fixed on entry to university, so a relationship here has an obvious causal direction. Causality is ambiguous when we examine whether students are similar to their alters in study habits: If they are, this could result from either homophily or diffusion.

**Centrality**

In addition to studying relationships between social ties, SNA uses ties to construct network-based individual attributes such as *centrality*. The centrality of a node is a measure of its connectedness to the remainder of the network (Carolan, 2013); as such, it is an intuitively natural construct but one that can be operationalized in numerous ways. The simplest is *degree*, in which, in the standard mathematical sense, the degree of a node is the number of its ties to other nodes.
Social Network Analysis (Borgatti & Everett, 2006). This can be considered a local measure. More complex, global measures include down-weighted counts of multistep links to more distant nodes (as in eigenvector or Katz centrality) or counts of the number of shortest paths around the network that pass through a given node (as in betweenness centrality; Borgatti, Carley, & Krackhardt, 2006). Some studies use multiple measures to assess different aspects of connectedness (e.g., Bruun & Brewe, 2013; Dou et al., 2016).

As with homophily, results on centrality can involve ambiguous causality—although people with high centrality might enjoy advantages, the mechanism by which these accrue is not obvious. If ties are conceptualized as providing influence over the network, then better connected people potentially have more influence (Brewe, Kramer, & Sawtelle, 2012). For instance, Hopkins, Ozimek, and Sweet (2017) found that coaches facilitated teachers’ implementation of a new mathematics curriculum by acting as brokers between the district office and the school, hence becoming catalysts for collective inquiry. If ties are conceptualized as providing access to resources, then better connected people can access more resources and, potentially, achieve better outcomes. For instance, Thomas (2000) found that better connected first-year students at a liberal arts college were less likely to drop out.

In our study, the resource-access interpretation makes more intuitive sense because we are interested in learning, which in our context is measured primarily by individual attainment. Considering the learning situation, however, highlights several mechanisms by which higher centrality might or might not be advantageous. Learning is not simple information acquisition, and students are not equal in their current knowledge or in their ability or desire to share it. So, having more alters might be beneficial, but this is not obvious. It could be that short conversations with numerous well-connected people provide the best access to mathematical information, solutions, conceptual understanding, and so on. Under this view, a measure such as eigenvector centrality might be appropriate. But it could be that individual understanding is better developed via longer conversations with smaller numbers of people. Under this view, which we consider more realistic in our situation, quality of ties is more important. Hence, we use degree as our preferred centrality measure (and, for some analyses, incorporate time spent collaborating as a distinct variable). But we note that better connectedness might nevertheless operate via multiple mechanisms. Students with higher centrality might be better connected to the activity of studying mathematics; they might therefore study more and perform better. But higher centrality might indicate a sociable, outgoing nature that co-occurs with a broad variety of time-consuming interests. So, students with higher centrality might study less and perform more poorly. In our study, we explored some of this complexity, investigating relationships between centrality, study habits, and mathematical performance. The results are reported after a discussion of relevant aspects of SNA methodology.

SNA Methodology

SNA studies typically take one of two basic approaches (Borgatti et al., 2013; Carolan, 2013). Egocentric studies use representative sampling to select people...
who will report on their individual social networks. Whole-cohort studies use data from every person in a specified group—as far as is practical—to construct a social network model for that group. We focus here on whole-cohort studies because a lecture class is a natural unit of investigation for questions about student–student interactions and because mathematics education researchers have not yet attempted to study such a class in this way. Our study involved collecting both individual attribute data, which is common to general quantitative studies, and data on ties, which is not. Moreover, the logic of whole-cohort studies is like that of case studies: Every network has different characteristics, and connections within the network are not mutually independent. This approach thus imposes demands and decisions that are distinctive to SNA, as listed below.

First, ethics must be carefully considered. In studies of individual attributes, participants report only about themselves. In studies of ties, individuals report about themselves and about specific others. The reported information could in theory be emotive or contentious. Participants asked about whom they like or trust might be unwilling to offer accurate reports (Borgatti et al., 2013). It can therefore be necessary to trade off detail against the likelihood of obtaining full participation and accurate data.

Second, the cohort needs to be defined. This consideration is common to all research but more salient in case studies, in which researchers must decide on the boundaries of the case and investigate a sufficient range and proportion of the individuals or activities involved (Yin, 2003). In SNA cohort studies, the consideration is tighter still because mapping a whole network ideally requires that everyone takes part (Carolan, 2013). This, of course, is unlikely to be practically manageable, and boundary issues are theoretical rather than absolute (Borgatti & Halgin, 2011; see also the Discussion section). But sufficient participation for a meaningful study requires effort to ensure that every potential cohort member can participate.

Third, participants should be supported in accurately reporting their ties. In small cohorts, they can be asked to check boxes on a roster, which reduces the risk that ties will be omitted (Borgatti et al., 2013). In larger cohorts, this might not be practical (though see Bruun & Brewe, 2013), so participants must be able to recall their interactions and state the names of their alters. Neither can be taken for granted. People do not have reliable memories and might, for instance, know their alters by nicknames or first names only. Ensuring that participants are informed in advance about what they will need to know is therefore prudent.

Fourth, the level of information to be gathered on each tie needs to be considered. Ties can be unweighted or weighted according to frequency or importance; they can be undirected or directed according to who seeks help from whom or reports to whom (Borgatti et al., 2013; Bruun & Brewe, 2013). Again, participants might benefit from advance information on what will be required.

Fifth, the consequences of missing data need to be considered. Again, this issue is common to all research but more salient in SNA. In SNA, if a person is absent or unwilling to participate, then their omission has consequences for mapping the network (Kossinets, 2006). This is especially true in small networks, where a missing node could result, for instance, in a network appearing to be broken into two separate clusters when in fact it is not (Borgatti et al., 2006). In larger
networks, dramatic alterations are less likely, but potential participants will probably be missing, and those who participate will probably forget some alters. As in all research, pragmatism is important because no measure is perfect. In SNA, although centrality measures are not exact, Borgatti, Carley, and Krackhardt (2006) report that they are fairly robust under small amounts of error.

Sixth, decisions are required about symmetry. If one person reports a tie that the alter does not, the researcher has a choice: work with directed links—allowing different weights to be attached to the link from A to B and that from B to A—or assume that interactions are more likely to be forgotten than invented and simplify by symmetrizing, taking the highest level of reported link to be accurate (Carolan, 2013). In this article, we take the symmetrizing approach.

Seventh, the “observations” captured in ties are inherently nonindependent, so the assumptions of typical hypothesis tests are violated. This does not prevent the researcher from testing hypotheses, and the logic remains the same: We ask how likely it is that a result as extreme as that found in the data would occur if there were no systematic relationship. But tests must be implemented using methods that do not make the standard statistical assumption of independence. The most common way to do this uses permutation methods, and this study took that approach, using quadratic assignment procedure (QAP) correlations and regressions implemented with the software package UCINET (Borgatti, Everett, & Freeman, 2002).

As a final note, visual representations of a network can be useful for providing intuition about and illustration of research results, and we provide these where appropriate below.

Method

United Kingdom, University, and Course Context

Because we are reporting a cohort study, we give a comparatively full description of the context.

In the United Kingdom, where this study took place, students intending to pursue higher education specialize to approximately three *A level* subjects at the age of 16, where passing grades are labelled A*, A, B, C, D, and E. At 18 (or later), those who go to university are accepted to study specific degree programs, usually either *single-honors* programs in which they study just one subject (perhaps with the possibility of “outside options”) or *joint-honors* programs in which they study two subjects, typically in a 50:50 or 75:25 ratio (more subjects are possible but rare). In mathematics, the first year of a program is often prescribed, so that all students attend the same lectures for the same set of approximately 12 courses. Because programs overlap, this means that lecture classes can be large: 200 students is common and by no means the largest. Many universities therefore operate a system of accompanying tutorials to provide more differentiated attention. Tutorials might function as problems classes for specific courses, involving perhaps 20–50 students, or as small-group tutorials offering support for all courses, with perhaps four to eight students. A mathematics support service might also be available (see Perkin & Croft, 2004). In later years, students usually have
some compulsory courses and some that they select from a range of options. Their lecture classes therefore tend to become smaller as they progress, but each course is offered only once per year, and the cohort progresses together.

Loughborough University, where this study took place, is campus based and offers on-campus accommodation in halls of residence to an unusually large proportion of its students; most of the remainder live nearby in shared houses or private halls of residence, and a small proportion commute from neighboring towns. The university has a strong sporting culture, with a disproportionate number of elite athletes and many students involved in sports at various levels. In other respects, including in its mathematics degrees, it is typical of comparably ranked universities, operating as described above. Each year it admits about 200 students to single-honors and joint-honors mathematics programs, with nominal required entry grades in the region of three A grades at A level. Actual entry grades vary: Some students arrive with considerably higher grades, perhaps with separate A levels in both Mathematics and Further Mathematics, and some are permitted to enter with lower grades. Mathematics and related courses are organized across two semesters, and each typically has three lectures or two lectures and one problems class per week, giving a total of 18 weekly contact hours. First-year students are able to access individual support via weekly academic tutorials involving six to eight students or via the drop-in service at the university’s two Mathematics Learning Support Centres (MLSCs). Students in later years do not have tutorials but can use the MLSCs in the same way.

The lecture course in which our data were collected was called Analysis 1. Of the 214 students registered (137 males and 77 females), approximately two thirds were first-year students on degree programs named Mathematics (89), Mathematics with Statistics (10), Mathematics with Economics (23), Financial Mathematics (21), Mathematics with Mathematics Education (7), or Mathematics and Management (4). The remainder were second-year students on degree programs named Mathematics and Sports Science (21), Mathematics and Accounting and Financial Management (19), Physics and Mathematics (8), or Computer Science and Mathematics (12). The course had three 50-minute lectures per week, during which the lecturer displayed partially populated notes on a visualizer and mixed traditional lecturing with short activities for students to complete in collaboration with their neighbors. The course was assessed via three 20-minute in-class tests (worth 10% each) and a 2-hour final examination (worth 70%). All students were exposed to different lecturing styles and assessment regimes in other courses. Lecture attendance was not monitored, and, by the time the data were collected, typical attendance in Analysis 1 (on days without in-class tests) was 160–170 students.

Data Collection

To maximize the number of students who were present to take the survey, two steps were taken. First, data collection took place in Week 9 of the 11-week term,
on the day of the third test, because students who are often absent do normally attend on test days. To minimize the chance that test stress would affect the reliability of responses, data were collected after the 20-minute test and a subsequent 10-minute break. Second, 3 weeks earlier, on the day of the previous test, the researchers were introduced, the study was explained, and students were shown the form they would be asked to complete and offered the opportunity to ask questions. They were informed, in particular, that to participate they would need to know the full names of those with whom they discussed mathematics.

On the day of data collection, participants were asked to complete a two-part survey. Part 1 asked them to report their percentage lecture attendance, number of hours per week spent on independent study, and number of hours within that spent on collaborative study. Throughout this article, we used these data to distinguish between collaborative independent study and individual independent study, which together make up total independent study. Part 2 asked them to list other students in the class with whom they spoke about mathematics frequently (once per week or more) and occasionally (less than once per week). We did not anticipate that reporting the fact rather than the content of a conversation about mathematics would make students uncomfortable (cf. Bruun & Brewe, 2013). Nevertheless, the attached informed consent form made clear that the raw data would be handled by a member of the research team who was unattached to the course and that if a student did not sign or return the form, any data provided about them by others would not be used. It also asked them to agree to the researchers obtaining demographic and attainment information from university databases. The survey instrument is available at https://doi.org/10.17028/rd.lboro.5505364.v1.

Data and Processing

Of the 214 registered students, 168 returned their forms, but 18 did not sign. We removed one further form because one student on a study-abroad program did not have the same background, range of courses, or available attainment information. Data from the remaining 149 students (90 males and 59 females) provided a total of 427 reported ties. In this report, we take two simplifying approaches. First, we treat all ties as equal, not differentiating between frequent and occasional ties. Second, we symmetrize, assuming that a missing tie has been forgotten rather than an extra one invented. Symmetrizing led to a total of 266 ties across the 149 participants.6

3 The second author of this paper handled the data processing; the first author was the course lecturer.

4 We took this maximally cautious approach but note that it results in more missing data. SNA researchers sometimes make different decisions, especially when longitudinal data are collected (e.g., Carolan, 2013; Huisman & Steglich, 2008).

5 Information on obtaining the anonymized data can be found at https://doi.org/10.17028/rd.lboro.5493136.v1.

6 If all 266 ties had been reciprocated, there would have been 266 × 2 = 532 reported ties. In our data, 532 – 427 = 105 ties were reported by one alter only.
Results

Who Studies With Whom?

Figure 1 shows a graph\(^7\) of the social network obtained from the data for the 149 participants, with first-year students in black and second-year students in gray (subsequent graphs retain the node layout to facilitate comparison). Table 1 summarizes the information in a different way by showing counts of participants with different degree centralities.

The distribution of degree centrality is wide, and the shading in Figure 1 suggests homophily: It appears, not surprisingly, that students primarily discuss mathematics with others in the same year. One further question, then, is whether homophily also operates at a lower level by program, as programs are subsets of year groups. Informal graph inspection indicated some possibility of this, and Figure 2 highlights nodes for students on two different programs, one first year and one second year. To investigate this possibility analytically, we constructed

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\(^7\) The graph layout is generated by UCINET using node repulsion and equal edge length bias. Consequently, the graph is comparatively readable, but edge lengths do not have interpretable meanings.
two matrices of 0s and 1s. One matrix (as is usual in SNA) captured the ties in the social network, where a 1 in place $m, n$ indicated a tie between participants $m$ and $n$. The other matrix was constructed so that 1 appeared in place $m, n$ if and only if participants $m$ and $n$ were on the same degree program. QAP correlation between these two matrices (Borgatti et al., 2013), implemented in UCINET,\(^8\) confirmed

\[^8\] All reported QAP correlations in this paper used 10,000 permutations.

Figure 2. Complete social network colored by program in two different ways. At the top, Mathematics with Mathematics Education in black; at the bottom, Mathematics and Accounting and Financial Management in black (all other participants appear in gray in both cases).
that the correlation between discussing mathematics and being on the same program was significant, \( r = .192, p < .001 \). This correlation is not high, reflecting the distribution of students to programs: We might expect that students on small degree programs would all know and speak to one another but that those on larger programs form less tight-knit groups.

One further question about homophily concerns gender, which is often important to mathematics educators (e.g., Mendick, 2006; Solomon, 2012) and which has been found to influence interactions in many situations\(^9\) (McPherson et al., 2001). Figure 3 shows the network with women in black. Both the graph and the statistics show that, in this cohort, gender homophily is not an important factor in who studies with whom: The women are spread across the network, and the QAP correlation between discussing mathematics and having the same gender is \( r = .011, p = .135 \), very small and not significant.

Finally, homophily could operate through living and sporting or social arrangements: Students might be disproportionately likely to study with those they live with or socialize with in sporting or other leisure activities. Our survey requested this information only for listed ties, so we do not know about students who share a hall or society but do not discuss mathematics. This means that we cannot repeat the statistical analyses, but Figure 4 shows the percentages of the 266 symmetrized ties for which the students shared a program, a hall or house, and a sport or social activity (plus combinations thereof). This shows that only small proportions of mathematical discussions occur between students who know one another entirely from other social settings; 86% of ties are associated with program.

\(^9\) Note that, according to McPherson et al. (2001), “race and ethnicity creates the strongest divides in our personal environments, with age, religion, education, occupation, and gender following in roughly that order” (p. 415).
Overall, then, there is evidence that a small proportion of ties are associated with accommodation and social activities, no evidence that ties are systematically associated with gender, and strong evidence that they are influenced by degree program.

Relationships Among Study Habits

In this section we address the questions: What are students’ study habits, and are these systematically related to the habits of those with whom they interact? We begin with individual study habits.

There was wide variation in self-reported lecture attendance and independent study time. Reported lecture attendance ranged from 20% to 100%, with median 90% and mean 85%. Total weekly hours of independent study ranged from 1 hour to 44 hours with median 10 hours and mean 13 hours. This means that for many students, total study time is lower than would be recommended by instructors (and the difference could be underestimated here if participants tended to overestimate their study time in their self-reports). The stated expectation is that each course should take 100 study hours over the semester, including lecture time; this equates to approximately 35 hours per week across all courses. Thus, weekly independent study time should be about 17 hours, around half a day to a day more than is typically happening. It is worth noting that this is not unusual. Time-diary studies in Germany and the United States have reported similar results (e.g., Babcock & Marks, 2011). Figure 5 shows a graph of total independent study time against lecture attendance. The correlation between the two was $r = .314$, $p < .001$; although we see a loose trend for students to be more or less “hard workers,” independent study time shows considerable variation even for those with high lecture attendance.

Wide variation was also apparent in participants’ allocation of their independent study time to individual and collaborative work. Reported weekly collaborative independent study ranged from 0 hours to 15 hours with median 2 hours and mean 3 hours. The graph in Figure 6 shows weekly collaborative hours against total weekly independent study hours; students reported spending up to half of their
independent study time collaborating. Students who study longer have more capacity to collaborate, but there was no clear relationship beyond that. Percentage\(^{10}\) of independent study time spent collaborating was not systematically related to total independent study time \((r = -.081, p = .324)\). This means that there is no evidence that students who collaborate more study for longer or that students trade off collaborative and individual study time.

\(^{10}\)Note that the graph in Figure 6 shows absolute numbers of collaborative hours but we use percentages here to investigate the relationship between total independent study time and proportion of that time spent collaborating.
We next relate study time to network data. Figure 7 shows the network graph for the 149 participants, with node area proportional to total independent study time. This graph indicates no obvious relationship between being better connected and studying for more hours—large nodes do not seem systematically connected to other large nodes. Statistical analyses confirm this observation. Degree centrality was significantly correlated with hours spent in collaborative independent study ($r = .434$, $p < .001$), meaning that those who study with more different people also tend to spend more time collaborating. But degree was not, in our data, significantly correlated with total independent study time ($r = .109$, $p = .190$); those who study with more peers do not systematically study for longer.

Finally, we consider the possibility of homophily and diffusion, asking whether individuals’ behaviors are systematically related to those of their alters. Again Figure 7 presents no obvious evidence of a relationship, and again this is confirmed by an analytical approach. We constructed a matrix with entry $(m, n)$ equal to the absolute difference of the number of independent hours of study between students $m$ and $n$. If homophily were present, we would expect these differences to be smaller where participants were alters and larger where they were not. In other words, we would expect a negative correlation between this and the adjacency matrix, which has a 1 in entry $(m, n)$ if a tie was reported and a 0 otherwise. The observed correlation was very slightly negative but not significant: $r = -.015$, $p = .084$. The graph in Figure 8 captures this lack of homophily in a different way, showing at most a very loose relationship between a connected individual’s independent study hours and the average of their alters’ total independent study hours.

Overall, these analyses show no straightforward systematic relationships among study habits of individuals and study habits of those with whom they interact; study habits vary widely, and homophily and diffusion do not seem meaningfully
evident. We note, however, that this leaves open the possibility that less straightforward social organizations might operate in a large class. One suggestion is that there might be “good students” to whom others go for advice. We cannot address this directly because we did not ask about advice-seeking behaviors, but we note that this is an interesting open question and that if ties are indeed more significant to one person than to an alter, such a phenomenon might account for some unreciprocated ties. We do, however, observe that it seems unlikely that students have a clear idea of one another’s study habits. The size of the class means that the variety we see here will be invisible to most students. Indeed, the lack of relationship between a student’s study habits and those of the student’s alters means that even students who study together might not be able to estimate what friends are doing based on their own behavior. We take up this point in the Discussion.

What Predicts Attainment?

Our final question is perhaps the most important from an educational perspective: Do study habits predict attainment? Addressing this question required further data processing because of the complex composition of the class; we explain this and our analyses below.

Because the class included both first- and second-year students, we constructed overall attainment scores by averaging final marks (assigned as percentages) from shared core courses: For first years, these were called Analysis 1 (the course in which the study was conducted), Methods 1, and Linear Algebra; for second years, they were Analysis 1 and Methods 3. These scores were converted within groups to $z$-scores to obtain comparable attainment scores for each individual relative to

![Figure 8. Alters’ average total independent study against total independent study.](image)
their year group. To investigate whether study habits predict attainment, we then conducted a QAP regression\textsuperscript{11} predicting attainment score from total independent study time, percentage of independent study time collaborating, and degree centrality (as these are potentially disjoint aspects of study behavior). This yielded a model that explained only 3\% of the variance in attainment ($F=1.526$, $p=.235$, $R^2 = .031$). To check whether this obscured different relationships between study habits and attainment for students who collaborate to different extents, we repeated this regression with an additional interaction term, total independent study time × percentage of independent study time collaborating. This again showed no relationship between study habits and attainment ($F=1.183$, $p=.385$, $R^2 = .032$).

This somewhat surprising result prompted us to check whether our attainment score behaved as expected in other respects. Because we would expect an association between prior attainment and this score, we constructed a prior attainment score based on incoming A-level grades. This was not straightforward because students can study for different numbers of A levels (three is typical, but four is not uncommon, and five is possible), because some students also have AS levels (half A levels), because choices of A levels are broad, and because some students study for separate A levels in both Mathematics and Further Mathematics. However, for university entrance purposes, grades are commonly converted to numerical scores\textsuperscript{12} (A* in an A level corresponds to 140 points, A to 120, B to 100, etc.; in an AS level there is no A*, A corresponds to 60 points, B to 50 points, etc.). To create a single measure of prior attainment that valued mathematics over general attainment, we added the full scores for any mathematics A levels and half of the scores for any other A levels. We also removed students for whom A-level grades were unavailable, leaving 143 participants in this analysis. Because this prior attainment score could be more predictive for first years than for second years (who have had a further intervening year of study), we then conducted a QAP regression predicting attainment score from prior attainment score, year group, and the interaction between the two. The resulting model was significant and predicted 16\% of the variance in attainment ($F=9.148$, $p=.007$, $R^2 = .165$). This result was driven by the effect of prior attainment; with the year-group and interaction terms removed, the resulting model had very similar properties ($F=27.650$, $p < .001$, $R^2 = .164$). We consider this to be as expected. Prior attainment is a significant predictor of undergraduate attainment, although the many changes in mathematics and in the learning situation mean that we would not expect it to explain a large proportion of the variance. This means that we do not have reason to doubt our attainment scores.

As a final check that our result on the lack of effect of study habits did not mask a more complex relationship, we conducted an analysis to capture how attainment deviates from expected attainment based on A-level scores. For this, we used as our independent variable the residuals from the immediately preceding analysis; the nonsignificance of the year group and interaction terms in that analysis means that there is no evidence that the relationship between attainment and prior

\textsuperscript{11} QAP regressions were implemented in UCINET using 10,000 permutations.

\textsuperscript{12} This conversion is correct for this cohort although the system has since changed.
attainment differs for first-year and second-year students, so these residuals provide a meaningful measure of attainment in relation to expected attainment based on A-level score. We conducted a QAP regression predicting these residuals from the same study habit variables used initially: total independent study time, percentage of independent study time spent collaborating, and degree centrality. This confirmed the overall result by yielding a model that was not significant ($F=2.036$, $p = .137$, $R^2 = .042$). We are thus confident to say that reported study habits were not systematically associated with attainment and did not have an effect over and above prior attainment. Possible reasons for this, along with a review of the results as a whole, are discussed below.

**Discussion**

**Summary**

The research reported here used SNA to address three questions in the context of undergraduate mathematics in a cohort-based higher education system:

1. Who studies with whom?
2. What are students’ study habits, and are these systematically related to the habits of those with whom they interact?
3. Do individual and collaborative study habits predict attainment?

In answer to Question 1, we found that who studies with whom is influenced primarily by program of study rather than by other characteristics such as gender or by other social interactions such as those arising from living arrangements or leisure activities. In answer to Question 2, we found that study habits varied widely and that students on average studied for less time than their department advises. We found a loose trend for students to be more or less hard working but also that independent study varied considerably even for those with high lecture attendance. Finally, we found that percentage of collaborative study was not systematically related to total independent study time, that those who studied with more different peers also tended to spend more time collaborating but not to study for longer overall, and that there was a significant but very weak association between an individual’s independent study hours and those of the individual’s alters. This variation and lack of systematic relationships mean that we consider it unlikely that students know a great deal about the study habits of their peers.

In answer to Question 3, we found no significant relationship between study habits and attainment, even when factoring out prior attainment. This finding is somewhat surprising, and we suggest that there are (at least) five possible explanations. First, there might be no systematic relationship to find. Perhaps study habits do not matter because success in undergraduate mathematics is primarily driven by other factors such as prior attainment or simply intelligence. Second, it could be that there is a systematic relationship, but we looked for it in the wrong place. Perhaps patterns of study are sufficiently erratic in first-year undergraduates—even 9 weeks into their first semester—that there is no discernible relationship at this stage, but there would be for more experienced students with more stable study routines and relationships. Third, study habits during term time could
be less important than those during the revision period (i.e., the preparation period immediately preceding examinations; for this class, the winter break and approximately the following 3 weeks). This thought is not palatable for instructors keen to promote consistent study, but it is certainly possible. Fourth, the amount of study and the extent to which it is collaborative might matter less than its more detailed contents: Perhaps the crucial question is how effectively students spend their time. This is plausible—studies have found that individual undergraduate mathematics learning can be improved by self-explanation training (Hodds, Alcock, & Inglis, 2014) and that engagement with different learning opportunities leads to different outcomes (Inglis et al., 2011). Finally, it could be that our study did not capture the interactions that matter. Perhaps our participants’ key interactions were with students outside the class, or with tutors, or with peers contacted online. Naturally, these possibilities raise questions for future research.

Research Implications

We consider research implications on two levels: questions for further research and the general utility of SNA in mathematics education.

If study habits really do not matter for success in undergraduate mathematics, then SNA might be helpful at a different scale from that used here, as in earlier studies of larger populations and more generic interactions (e.g., Thomas, 2000). It might be particularly useful for studying the effects of educational interventions, such as orchestration of living-learning communities (Kurotsuchi Inkelas, Daver, Vogt, & Brown Leonard, 2007; Stassen, 2003) and provision of subject-specific study spaces (Brewe, Kramer, & Sawtelle, 2012). If study habits do matter but we did not focus on the right variables, then SNA has much to offer. For instance, it could be that study habits change over a semester or an undergraduate program and that this is worth tracking longitudinally, both to investigate links to attainment and for intrinsic interest in what causes formation and dissolution of ties (cf. Dou et al., 2016; Rivera, Soderstrom, & Uzzi, 2010). If help-seeking behavior is important, then directed ties could be useful in investigating its effects on social networks and eventual outcomes. Similarly, social behaviors and outcomes might be linked to the contents of study time (Bruun & Brewe, 2013). We asked for no specifics about what our participants worked on during individual and collaborative study, but such information could be incorporated into future investigations.

Future work could also consider a different approach to cohort definition. We used a predefined lecture class, but, as noted above, students are likely also influenced by wider interactions. Methodologically, this is not problematic unless one takes a naive realist perspective, believing that there is a “true” network in the world and it is our job as researchers to discover it (Laumann, Marsden, & Prensky, 1983). As Borgatti and Halgin (2011) argue, a nominalist perspective is more appropriate: Boundary specification is properly thought of not as an empirical problem but as related to a research question and the associated explanatory theory. Here, we were interested in behaviors and relationships among a large lecture class. As a result, we did not aim to capture all interactions that every student might have. But other investigations might take different approaches or work with larger populations. SNA studies using electronic information have, for instance,
considered populations in the thousands to study college dropout (Thomas, 2000) and in the millions to study the spread of information online (Bakshy, Rosenn, Marlow, & Adamic, 2012). Such work, of course, raises different ethical issues, and researchers using any form of SNA must balance informed consent and confidentiality with likely willingness to participate. Our study took a cautious approach of asking directly for participation and removing all ties involving students who did not consent to their data being used. But this meant removing some ties reported by participating students, and alternative approaches are possible.

Overall, we believe that SNA might be considered underutilized in mathematics education. Our field is interested in social aspects of learning (Inglis & Foster, 2018; Lerman, 2000) but has evolved to a state that is effectively dichotomized: Researchers interested in social issues typically use qualitative methods and sometimes consider quantitative approaches insufficient to address the complexity of social situations; researchers interested in cognitive issues often use quantitative methods and sometimes consider qualitative approaches unconvincing. We argue that this is unfortunate and that SNA could provide a link between study of small-scale individual interactions and larger scale social behaviors. It can be combined with experimental designs in large-scale studies in online contexts (Bakshy et al., 2012; Centola, 2010). Researchers interested in personal study decisions might consider applying it in combination with analyses of personality, attitudinal, or self-efficacy factors or beliefs about the value of collaborative study (e.g., Dou et al., 2016). It can also be combined with more detailed analyses of the content of interactions (Lee & Bonk, 2016) and the social roles and capital acquired and maintained by individuals and organizations (Brewe et al., 2012; Rasmussen, Zandieh, & Wawro, 2010; Wenger, 1998). Researchers interested in students with particularly high or low numbers of alters or amounts of collaborative study time could follow up with more detailed qualitative analyses. We suggest that SNA could thus provide a focus around which researchers from different traditions could begin productive conversations.

Pedagogical Implications

Pedagogical implications of our work are, to put it mildly, less clear than we had hoped. Our results certainly do not make it possible to advise students confidently on which study habits are likely to be productive. One reading of this is that social networks are not important for undergraduate mathematics instructors—that other factors should claim instructors’ attention because they have more effect on outcomes. This is reasonable, but given the unexplored possibilities listed above, we suggest that it might also be hasty. We certainly think it noteworthy that study habits vary widely and that students might not be fully aware of this. Instructors might wish to use our results—or to gather their own information—to start a conversation with students about actual as opposed to nominally required study behavior. Students who spend only a couple of hours per week in independent study might be shocked to learn that some of their peers study for 10 times as long, and students who spend 40 hours per week might learn that they could be putting themselves under too much pressure. Everyone might wish to consider the benefits and drawbacks of study time spent collaborating.
Instructors might also wish to consider the likely generalizability of our work to their own contexts. We have provided a snapshot of study habits in a single (if large) class at a single university. For the U.K. context, the cohort can be considered fairly typical. It is in the low hundreds, all students have strong but not elite prior mathematical performance, they share most but not all of their lectures, and they are organized into smaller groups for tutorials. There is no a priori reason to believe that students in similar circumstances elsewhere would behave in dramatically different ways, though of course this is an empirical question—apparently similar universities might have different study cultures. Under more strikingly different circumstances, we might expect behavior to differ in relatively predictable ways. In city universities, for instance, or those in which more students live at home rather than on campus, we might expect who studies with whom to be influenced less by degree program and more by proximity of housing or shared social activities. In countries where the cohort model of higher education is not the norm, so that students are in smaller classes and do not all progress together, it might not make sense to think in terms of a bounded cohort at all, and studies might track other aspects of social behavior.

With this in mind, we conclude by returning to our opening point: Despite our field’s interest in social issues and instructors’ interest in their students’ interactions, research on those interactions has been largely if not exclusively confined to the classroom. Thus our study, with its combined focus on individual study behaviors and mathematically focused interactions, both contributes new understanding of mathematical learning behaviors and exemplifies how SNA can be applied in mathematics education.

References


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