Over the past decade, mathematical and statistical methods have become increasingly important for studying the properties and impact of political redistricting. Beyond supporting gerrymandering claims in litigation, these quantitative approaches have also proved useful for evaluating potential impacts of legislative reform efforts and helping to design new plans in the current cycle. From a mathematical perspective, the key idea is to formulate a discrete model of redistricting as a graph partitioning problem and then apply probabilistic methods, including MCMC as motivated by similar problems in statistical physics, to establish baselines and detect outliers. In this talk, I will present several applications of this methodology to detecting gerrymandering and discovering unintended consequences of "neutral" criteria. This will lead to a natural discussion of open problems in this research field, where I will describe several recent and ongoing projects that have included student researchers.