Analysis of Variance for a Randomized Block Design in Minitab

Example: An accounting firm, prior to introducing in the firm widespread training in statistical sampling for auditing, tested three training methods: (1) study at home with programmed training materials, (2) training sessions at local offices conducted by local staff, and (3) training session in Chicago conducted by a national staff. Thirty officers were grouped into ten blocks of three, according to time elapsed since college graduation, and the auditors in each block were randomly assigned to the three training methods. At the end of the training, each auditor was asked to analyze a complex case involving statistical applications; a proficiency measure based on this analysis was obtained for each auditor. The results were (block 1 consists of auditors graduated most recently, block 10 consists of those graduated most distantly):

<table>
<thead>
<tr>
<th>Block</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>73</td>
<td>81</td>
<td>92</td>
</tr>
<tr>
<td>2</td>
<td>76</td>
<td>78</td>
<td>89</td>
</tr>
<tr>
<td>3</td>
<td>75</td>
<td>76</td>
<td>87</td>
</tr>
<tr>
<td>4</td>
<td>74</td>
<td>77</td>
<td>90</td>
</tr>
<tr>
<td>5</td>
<td>76</td>
<td>71</td>
<td>88</td>
</tr>
<tr>
<td>6</td>
<td>73</td>
<td>75</td>
<td>86</td>
</tr>
<tr>
<td>7</td>
<td>68</td>
<td>72</td>
<td>88</td>
</tr>
<tr>
<td>8</td>
<td>64</td>
<td>74</td>
<td>82</td>
</tr>
<tr>
<td>9</td>
<td>65</td>
<td>73</td>
<td>81</td>
</tr>
<tr>
<td>10</td>
<td>62</td>
<td>69</td>
<td>78</td>
</tr>
</tbody>
</table>

INPUTTING DATA:

<table>
<thead>
<tr>
<th>TREATMENT</th>
<th>BLOCK</th>
<th>SCORE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>73</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>81</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>92</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>76</td>
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<tr>
<td>2</td>
<td>2</td>
<td>78</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>89</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

COMMANDS IN MINITAB: STAT > ANOVA > General Linear Model > RESPONSE > SCORE
MODEL > TREATMENT BLOCK
COMPARISONS > Pairwise comparisons > Terms > TREATMENT > OK
GRAPH > Histogram of residuals, normal plot of residuals, residuals vs. fits > OK > OK
GRAPH > Scatterplot > With Groups > y-variable SCORE > X-VARIABLE TREATMENT > Categorical Variable BLOCK > Data View > Connect Line OK > OK > OK

ASSESSING THE REASONABLENESS OF THE NORMALITY ASSUMPTION:
ASSESSING THE REASONABLENESS OF THE EQUAL VARIANCE ASSUMPTION:

ASSESSING THE APPROPRIATENESS OF THE NO INTERACTION ASSUMPTION:

Interpretation: From the appearance of the graphs, the normality and equal variance assumptions are reasonable. There is some concern, however, about the appropriateness of the no interaction assumption. Now we perform an analysis for a randomized block design.

Minitab Output for Performing Analysis for Randomized Block Design:

Analysis of Variance for score, using Adjusted SS for Tests

<table>
<thead>
<tr>
<th>Source</th>
<th>DF</th>
<th>Seq SS</th>
<th>Adj SS</th>
<th>Adj MS</th>
<th>F</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>treat</td>
<td>2</td>
<td>1295.00</td>
<td>1295.00</td>
<td>647.50</td>
<td>103.75</td>
<td>0.000</td>
</tr>
<tr>
<td>block</td>
<td>9</td>
<td>433.37</td>
<td>433.37</td>
<td>48.15</td>
<td>7.72</td>
<td>0.000</td>
</tr>
<tr>
<td>Error</td>
<td>18</td>
<td>112.33</td>
<td>112.33</td>
<td>5.64</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>29</td>
<td>1840.70</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

S = 2.49815   R-Sq = 93.90%   R-Sq(adj) = 90.17%
Unusual Observations for score

<table>
<thead>
<tr>
<th>Obs</th>
<th>score</th>
<th>Fit</th>
<th>SE Fit</th>
<th>Residual</th>
<th>St Resid</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>76.000</td>
<td>71.833</td>
<td>1.580</td>
<td>4.1667</td>
<td>2.15 R</td>
</tr>
<tr>
<td>15</td>
<td>71.000</td>
<td>75.833</td>
<td>1.580</td>
<td>-4.8333</td>
<td>-2.50 R</td>
</tr>
</tbody>
</table>

R denotes an observation with a large standardized residual.

\[ H_0: \mu_1=\mu_2=\mu_3 \]

\[ H_a: \text{At least two of the treatment means are different.} \]

\( \alpha = 0.05 \)

\( F = 103.75 \)

p-value = .000 < \( \alpha \) thus reject \( H_0 \)

**Conclusion:** The data provide sufficient evidence to conclude that at least two of the treatment means are different. Now we will apply Tukey’s multiple comparisons procedure to locate which treatment means are different.

Grouping Information Using Tukey Method and 95.0% Confidence

<table>
<thead>
<tr>
<th>treat</th>
<th>N</th>
<th>Mean</th>
<th>Grouping</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>10</td>
<td>86.1</td>
<td>A</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>74.6</td>
<td>B</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>70.6</td>
<td>C</td>
</tr>
</tbody>
</table>

Means that do not share a letter are significantly different.

Tukey 95.0% Simultaneous Confidence Intervals
Response Variable score
All Pairwise Comparisons among Levels of treat
treat = 1 subtracted from:

| treat | Lower | Center | Upper | --------+---------+---------+--------|
|-------|-------|--------|-------|---------|---------|---------|
| 2     | 1.148 | 4.000  | 6.852 | (-----*-----) |
| 3     | 12.648| 15.500 | 18.352| (------*-----) |

5.0 10.0 15.0


treat = 2 subtracted from:

| treat | Lower | Center | Upper | --------+---------+---------+--------|
|-------|-------|--------|-------|---------|---------|---------|
| 3     | 8.648 | 11.50  | 14.35 | (-----*-----) |

5.0 10.0 15.0