**t Distribution**

When $\bar{x}$ is the mean of a random sample of size $n$ from a normal distribution with mean $\mu$, the random variable

$$
t = \frac{\bar{x} - \mu}{s/\sqrt{n}}
$$

has a probability distribution called a *t distribution* with $n - 1$ degrees of freedom (df).

**Properties of t Distributions**

Let $t_v$ denote the density function curve for $v$ df.

1. Each $t_v$ curve is bell-shaped and centered at 0.
2. Each $t_v$ curve is more spread out than the standard normal ($z$) curve.
3. As $v \to \infty$, the sequence of $t_v$ curves approaches the standard normal curve (so the $z$ curve is often called the $t$ curve with $df = \infty$)

![Graph showing t-distribution with 3 df compared to standard normal distribution]

**Historical Perspective**

The distribution of the $t$ statistic in repeated sampling was discovered by W. S. Gosset, a chemist in the Guinness brewery in Ireland, who published his discovery in 1908 under the pen name of Student.
Confidence Interval for $\mu$
Using a Small Sample ($n \leq 30$)

\[
\bar{x} \pm t_{\frac{\alpha}{2}, n-1} \frac{s}{\sqrt{n}}
\]

Assumptions:
1. A simple random sample is selected from the population
2. The population distribution is approximately normal.

Hypothesis Test for $\mu$
Using a Small Sample ($n \leq 30$)

$H_0$: $\mu = \mu_0$

$H_a$: 1. $\mu > \mu_0$
2. $\mu < \mu_0$
3. $\mu \neq \mu_0$

Test Statistic:

\[
t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}
\]

Rejection Region: For a probability $\alpha$ of a Type-I error, we can reject $H_0$ if

1. $t > t_\alpha$
2. $t < -t_\alpha$
3. $t > -t_{\alpha/2}$ or $t < t_{\alpha/2}$

Assumptions:
1. A simple random sample is selected from the population
2. The population distribution is approximately normal.
Example  The owners of a large real estate agency believe that a slow economy has lowered the selling prices of homes below last year’s average of $102,000. A random sample of the selling prices of 18 homes sold (expressed in thousands of dollars) reported the following figures:

105.0  104.0  81.0  128.0  74.0  111.0  86.9  87.9  87.0  84.0  92.0  92.9  108.0  85.9  115.0  134.0  84.5  91.9

Do the data provide sufficient evidence to conclude that the average selling price of homes sold by this firm has decreased. Conduct hypothesis test using $\alpha = .01$.

Assessing Reasonableness of Normality Assumption Using Minitab

Minitab Commands for Normal Probability Plot: `> stat > basic statistics > normality test`

Conducting Hypothesis Test Using Minitab

Minitab Commands: `> stat > basic statistics > 1-sample t > test mean 102 > options > alternative less than`

Minitab Output:

Test of mu = 102 vs < 102

<table>
<thead>
<tr>
<th>Variable</th>
<th>N</th>
<th>Mean</th>
<th>StDev</th>
<th>SE Mean</th>
<th>95% Upper Bound</th>
<th>T</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>C10</td>
<td>18</td>
<td>97.3889</td>
<td>16.5696</td>
<td>3.9055</td>
<td>104.1829</td>
<td>-1.18</td>
<td>0.127</td>
</tr>
</tbody>
</table>
Constructing Confidence Interval Using Minitab

**Example** A survey was conducted to estimate $\mu$, the mean salary of middle-level bank executives. A random sample of 15 executives yielded the following yearly salaries (in units of $1000$):

<table>
<thead>
<tr>
<th>Salary (in $1000$)</th>
<th>88</th>
<th>121</th>
<th>75</th>
<th>39</th>
<th>52</th>
<th>102</th>
<th>95</th>
<th>78</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>69</td>
<td>82</td>
<td>80</td>
<td>84</td>
<td>72</td>
<td>115</td>
<td>106</td>
<td></td>
</tr>
</tbody>
</table>

Find a 98% confidence interval for $\mu$. Interpret the interval!

Assessing Reasonableness of Normality Assumption Using Minitab

**Constructing Confidence Interval Using Minitab**

**Minitab Commands:**

```plaintext
> stat > basic statistics > 1-sample t > options > confidence level > 98 > alternative not equal
```

**Minitab Output:**

**One-Sample T: C19**

<table>
<thead>
<tr>
<th>Variable</th>
<th>N</th>
<th>Mean</th>
<th>StDev</th>
<th>SE Mean</th>
<th>98% CI</th>
</tr>
</thead>
<tbody>
<tr>
<td>C19</td>
<td>15</td>
<td>83.8667</td>
<td>22.0871</td>
<td>5.7029</td>
<td>(68.8996, 98.8338)</td>
</tr>
</tbody>
</table>