Graded HW 1 is coming around.

Checked these:  2.1  6  2 pts
                2.1  33  3 pts
                2.2  18  2 pts
                2.2  15  3 pts

You are always expected to show some work or explanation.

Solutions to the above probs. are posted at my math 216 webpage.
Consider the statement:

All pigs that can fly live in Australia.

Symbolically:

\[ P = \text{set of all pigs} \]

\[ F(x) \text{ mean } p \in P \text{ can fly} \]

\[ A(x) \text{ mean } x \text{ lives in Australia} \]

\[ \forall x \in P, \ F(x) \rightarrow A(x) \]

Negation:

\[ \neg (\forall x \in P, \ F(x) \rightarrow A(x)) \]

\[ \equiv \exists x \in P \text{ such that } \neg (F(x) \rightarrow A(x)) \]

\[ \equiv \exists x \in P \text{ such that } F(x) \land \neg A(x) \]
In words: There is a pig that can fly and does not live in Australia.

Clearly false, so original statement must be true.

In general: The statement

$$\forall x \in D, \ p(x) \rightarrow q(x)$$

is considered true if \( p(x) \) is false for all \( x \in D \) "Vacuously"
3.3 Statements involving multiple quantifiers

Example: \( \forall x \in \mathbb{R}^+ \)

\( \exists x \in \mathbb{R}^+ \text{ such that } \forall y \in \mathbb{R}^+, \, x \leq y \).

More formally: \( \exists x \in \mathbb{R}^+ (\forall y \in \mathbb{R}^+ (x \leq y)) \)

In words: there is a smallest positive real

Negation: \( \neg (\exists x \in \mathbb{R}^+ (\forall y \in \mathbb{R}^+ (x \leq y))) \)

\( \equiv \forall x \in \mathbb{R}^+ (\neg (\forall y \in \mathbb{R}^+ (x \leq y))) \)
\[\forall x \in \mathbb{R}^+ \quad (\exists y \in \mathbb{R}^+ \quad (\neg (x \leq y)))\]

\[\equiv \forall x \in \mathbb{R}^+, \exists y \in \mathbb{R}^+ \text{ such that } y < x.\]

In words: For every positive real \( x \), there is a smaller positive real.

Eventually:

\[\neg (\forall x \in D, \exists y \in D \text{ such that } P(x, y))\]

\[\equiv \exists x \in D \text{ such that } \forall y \in D, \neg P(x, y)\]

\[\neg (\forall x \in D, \forall y \in D, P(x, y))\]

\[\equiv \exists x \in D \quad (\exists y \in D \quad (\neg P(x, y)))\]
Note: with $\forall, \exists$ and $\exists, \forall$, the order does not matter:

$\forall$ people $x$, $\exists$ person $y$ s.t. $x$ loves $y$.

$\exists$ person $y$ s.t. $\forall$ people $x$, $x$ loves $y$.

3.3(9) $D = \{-2, -1, 0, 1, 2\} = E$

Explain why true:

a) $\forall x \in D, \exists y \in E$ s.t. $x + y = 0$.

This says that for any $x$ chosen from $D$, there is another element $y$ in $E$ so that $x + y = 0$. 
This is true because:
- if \( x = -2 \), use \( y = 2 \)
- if \( x = -1 \), use \( y = 1 \)
- if \( x = 0 \), use \( y = 0 \)
- if \( x = 1 \), use \( y = -1 \)
- if \( x = 2 \), use \( y = -2 \).

3.3 (3) Everybody loves somebody.

\[ \forall \text{people } x, \exists \text{ person } y \text{ s.t. } x \text{ loves } y. \]

or \( P = \text{all people}, \ L(x,y) \text{ means } x \text{ loves } y, \)
so:
\[ \forall x \in P \left( \exists y \in P \left( L(x,y) \right) \right) \]

(10) d) \( \forall x \in \mathbb{R}^+, \exists y \in \mathbb{R}^+ \text{ s.t. } xy = 1. \)

True. For any chosen \( x \in \mathbb{R}^+ \), \( y = \frac{1}{x} \) works:
\[ x \left( \frac{1}{x} \right) = 1, \]
3.4 Arguments involving quantifiers.

The rule of *universal instantiation* (in-stan-she-AY-shun) says the following:

If some property is true of *everything* in a set, then it is true of *any particular* thing in the set.

Can combine this idea with previous argument forms, like this:

\[
\text{Modus Ponens} \quad \begin{align*}
\text{universal Modus Ponens} \\
\forall x, \ P(x) \to Q(x) \end{align*} \\
\begin{align*}
P(a) \\
\hline
Q(a)
\end{align*}
\]

To be continued.