MML HW 1a is 12 orientation questions, and questions like:

5.1 #4; 5.2 #1, 3, 5, 7, 8; 5.3 #1, 2, 3, 5, 9, 24, 25, 29, 31;

5.4 #1, 2, 3; 5.5 #2, 3, 5, 6, 13, 14, 16, 17, 18, 39, 40.

Due by Mon, Jan 10.

MML HW 2a (due Thurs 1/21) is like:

6.2 # 3, 4, 5, 8, 9, 15, 16, 27, 33, 35, 52, 59;

6.3 # 1, 9

My office hrs: Neill 315, Tu, Th noon-2 and by appointment.
6.2 Finding Areas, like:

Know:

\[ y = f(x) \]

\[ \text{area} = \int_{a}^{b} f(x) \, dx \]
In a situation like these:

\[ y = f(x) \]
\[ y = g(x) \]

Can describe the region as
\[ a \leq x \leq b, \quad \text{and} \quad g(x) \leq y \leq f(x) \]

or: upper curve is \( y = f(x) \)
lower is \( y = g(x) \),
\( x \) goes \( a \) to \( b \).

So area is \( \int_{a}^{b} (f(x) - g(x)) \, dx \).
\[(\text{equiv. to} \quad \int_{a}^{b} f(x) \, dx - \int_{a}^{b} g(x) \, dx)\]

Here:

\[\begin{align*}
&\quad f(y) - g(y) \\
&\quad \text{Area} = \int_{c}^{d} (f(y) - g(y)) \, dy
\end{align*}\]
Here:

The total area is given by:

\[ \text{total area} = \int_a^c (f(x) - h(x)) \, dx \]

\[ + \int_c^b (g(x) - h(x)) \, dx \]
Area = \int_{c}^{b} (g(y) - h(y)) \, dy + \int_{b}^{d} (f(y) - h(y)) \, dy
Examples
6.2 (8)

\[
\text{Area} = 2 \int_0^b \left( 4 \cos^2(x) - \frac{1}{4} \sec^2(x) \right) dx
\]

To find \( b \):
\[
\frac{1}{4} \sec^2(x) = 4 \cos^2(x)
\]

\[
\Rightarrow \frac{1}{4 \cos^2(x)} = 4 \cos^2(x)
\]

\[
\Rightarrow \cos^4(x) = \frac{1}{16}
\]
\[
\cos(x) = \pm \frac{1}{2} \Rightarrow x = \cos^{-1}\left(\frac{1}{2}\right)
\]

\[
\Rightarrow x = \frac{\pi}{3}
\]

So Area = \[2 \int_{0}^{\frac{\pi}{3}} \left[ 4 \cos^2(x) - \frac{1}{4} \sec^2(x) \right] dx \]

To find \[\int \cos^2(x) \, dx,\]
use \[\cos^2(x) = \frac{1}{2}(1 + \cos(2x))\]

\[
= 2 \left[ \frac{2x + \sin(2x)}{2} - \frac{1}{4} \tan(x) \right]_0^{\frac{\pi}{3}} = \ldots
\]
\[ \int_{0}^{b} (\sin(2x) - \sin(x)) \, dx + \int_{b}^{\pi} (\sin(x) - \sin(2x)) \, dx. \]

To get \( b \):

\[ \sin(x) = \sin(2x) \]

\[ \sin(x) = 2 \sin(x) \cos(x) \]

\[ 0 = 2 \sin(x) \cos(x) - \sin(x) \]
0 = \sin(x) \left[ 2 \cos(x) - 1 \right]

so \ \sin(x) = 0 \ or \ 2 \cos(x) - 1 = 0

\Rightarrow x = \ldots, -\pi, 0, \pi, \ldots

\Rightarrow \cos(x) = \frac{1}{2}

\Rightarrow x = \ldots, \frac{\pi}{3}, \frac{2\pi}{3}, \ldots

\begin{align*}
\text{25} & \quad \text{region bounded by} \quad x = y^2 - 3y + 12 \\
& \quad \text{and} \quad x = -2y^2 - 6y + 30 \\
\text{Area:} & \quad \int \left( -2y^2 - 6y + 30 \right) - \left( y^2 - 3y + 12 \right) \ dy \\
& \quad \int \left( -3y^2 - 3y + 18 \right) \ dy
\end{align*}
to find limits of $f$:

$$y^2 - 3y + 12 = -2y^2 - 6y + 30$$

$$\Rightarrow \quad 3y^2 + 3y - 18 = 0$$

$$\Rightarrow \quad 3(y^2 + y - 6) = 0$$

$$\Rightarrow \quad 3(y + 3)(y - 2) = 0 \Rightarrow y = -3, 2$$

10. **Area** = $$\int_{-3}^{2} (-3y^2 - 3y + 16) \, dy = \cdots$$

$$\left[ -y^3 - \frac{3}{2}y^2 + 18y \right]_{-3}^{2} = \cdots$$