6.1 Net Change.

Main Idea:

\[
\int_{a}^{b} f'(x) \, dx = f(b) - f(a)
\]

\(\text{net change in } f(x)\)

When object moves along an axis:
\[ s'(t) = v(t) = \text{velocity} \]

\[ s''(t) = v'(t) = a(t) = \text{acceleration} \]

So:

\[ \int_{a}^{b} v(t) \, dt = \int_{a}^{b} s'(t) \, dt = s(b) - s(a) \]

Net area under \( v(t) \), \( t = a \) to \( t = b \).

Net change in position, \( t = a \) to \( t = b \).

"Displacement".

Or:

\[ \int_{a}^{t} v(x) \, dx = s(t) - s(a) \], which leads to:
6.1 (19) Given: \( v(t) = 9 - t^2 \), \( s(0) = -2 \).

I want to find a formula for \( s(t) \).

**Option 1:**

\[
\begin{align*}
\int v(t) \, dt &= \int (9 - t^2) \, dt \\
&= 9t - \frac{1}{3} t^3 + C
\end{align*}
\]

\[ s(0) = -2 \implies 9(0) - \frac{1}{3}(0)^3 + C = -2 \]

\( \implies C = -2 \), so

\[
\boxed{s(t) = 9t - \frac{1}{3}t^3 - 2}
\]
Option 2: \[ s(t) = s(a) + \int_a^t v(t) \, dt \]

\[ = s(0) + \int_0^t v(x) \, dx \]

\[ = -2 + \int_0^t (9 - x^2) \, dx \]

\[ = -2 + \left[ 9x - \frac{1}{3}x^3 \right]_0^t \]

\[ = -2 + 9t - \frac{1}{3}t^3 \]

**Note:** when an object moves along an axis, "displacement" and "distance travelled" are different, in general.
Ex. 6.1 (2) \( v(t) = -t^2 + 5t - 4 \).

Note: \( v(t) = -(t^2 - 5t + 4) \)

\[ = -(t-4)(t-1) \]

\[ t = 1 \quad t = 4 \]

6) displacement from \( t=0 \) to \( t=5 \) is

\[ s(5) - s(0) = \int_0^5 v(t) \, dt = \int_0^5 (-t^2 + 5t - 4) \, dt \]

\[ = \left[ -\frac{1}{3}t^3 + \frac{5}{2}t^2 - 4t \right]_0^5 = \frac{-125}{3} + \frac{125}{2} - 20 = \square \]
c) Distance traveled from $t=0$ to $t=5$ is:

$$\left| \int_0^4 v(t) \, dt \right| + \left| \int_4^5 v(t) \, dt \right| + \left| \int_4^5 v(t) \, dt \right|$$

$$= (\quad) + (\quad) + (\quad) = 0.$$

Generically, suppose $v(t)$ (velocity) looks like

This:

Displacement of object from $t=a$ to $t=b$ is

$$\int_a^b v(t) \, dt = \text{net area under } v(t) \text{ from } a \text{ to } b.$$
distance traveled from \( t = a \) to \( t = b \) is

\[
\left| \int_a^c v(t) \, dt \right| + \left| \int_c^b v(t) \, dt \right|
\]

equivalent to

\[
\int_a^b |v(t)| \, dt
\]