Math 171 Fall 2014 Final Exam

December 15, 2014

Last Name: ___________________________________________________________
First Name: ___________________________________________________________
Student ID: ___________________________________________________________
Section: ___________________________________________________________

Remember to show all of your work and provide all necessary explanations for full credit. Good luck and have a good break!

<table>
<thead>
<tr>
<th>Question</th>
<th>1</th>
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Question 1. (4 points each) Compute the following derivatives.

A. \( \frac{d}{dx} \left( 5x^3 - 2 - \frac{3}{\sqrt{x}} + \frac{1}{x^2} \right) \)

B. \( \frac{d}{d\theta} (\sin \theta - \cos \theta + \tan \theta - \sec \theta) \)

C. \( \frac{d}{dx} (e^x + \ln x - \tan^{-1} x + \sin^{-1} x) \)

D. \( \frac{d}{dx} \left( (x^3 + 4x^2)e^x \right) \)

E. \( \frac{d}{dx} \left( \frac{\sin x + \cos x}{x^2 + 1} \right) \)

F. \( \frac{d}{dx} (\ln(e^{2x} - \tan x)) \)
Question 2. (6 points each) Compute the following indefinite integrals.

A. \( \int (2x^3 - 3 + x^{-1} - x^{-3}) \, dx \)

B. \( \int x^2 \sec^2(x^3) \, dx \)

Question 3. (7 points each) Compute the following definite integrals. You are not required to simplify your answers.

A. \( \int_0^{\pi/2} (x^2 + \sin x) \, dx \)

B. \( \int_1^e \frac{(\ln x)^2}{x} \, dx \)
Question 4. (4 points each) Consider the function $f(x) = \frac{1}{4} x^4 - \frac{1}{3} x^3 - \frac{1}{2} x^2 + x - 1$. For reference, $f'(x) = (x + 1)(x - 1)^2$ and $f''(x) = (x - 1)(3x + 1)$. For full points on each part, justification must be provided.

A. List the critical point(s) of $f$.

B. On what interval(s) is the function $f$ decreasing?

C. For each critical point, state whether it is a local minimum, a local maximum or neither.

D. On what interval(s) is $f$ concave up?

E. List the inflection points(s) of $f$. 
Question 5. (20 points) Draw a graph of \( y = f(x) \) on the grid below that passes through the points \((-1,0), (0,1), (1,0), \) and \((2, -1)\) indicated by black dots on the grid. Your graph for \( f \) must be continuous and satisfy the following sign data for \( f' \) and \( f'' \). Also assume \( f'(0) \) and \( f''(0) \) do not exist.

<table>
<thead>
<tr>
<th>Interval</th>
<th>( f'(x) )</th>
<th>( f''(x) )</th>
</tr>
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<tbody>
<tr>
<td>((-\infty, -1))</td>
<td>Negative</td>
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</tr>
<tr>
<td>((-1, 0))</td>
<td>Positive</td>
<td>Positive</td>
</tr>
<tr>
<td>((0, 1))</td>
<td>Negative</td>
<td>Negative</td>
</tr>
<tr>
<td>((1, 2))</td>
<td>Negative</td>
<td>Positive</td>
</tr>
<tr>
<td>((2, \infty))</td>
<td>Positive</td>
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Question 6. (20 points) A rectangular sheet of cardboard of width $4w$ and height $h$ (in inches) can be folded into quarters and joined at the ends to make a “square tube” of volume $V = w^2h$ as drawn below:

The manufacturer of the cardboard sheet insists that $h$ be no more than 16 inches and $w$ be no more than 4 inches. What are the values of $h$ and $w$ so that the volume of the square tube is 16 in$^3$ and the quantity $P = w + \frac{1}{4}h$ is minimized?

For full credit, you must justify that you have in fact minimized, rather than maximized, the value of $P$. 
Question 7. (5 points) Use linear approximation to estimate \( \ln(1.05) \). (NOT ON 2015 EXAM)
Question 8. (5 points) An object’s position function is $s(t)$ and its velocity function is $v(t) = s'(t)$. If $s(1) = 3$ and $s(4) = 15$, the mean value theorem guarantees that the velocity of the object must be what value at some time between $t = 1$ and $t = 4$? For full credit, explicitly show the computation that produces your result.
Question 9. (10 points) Find the absolute minimum and absolute maximum values along with their locations attained by $f(x) = x^4 - 2x^2 + 4$ on the interval $[-1,2]$.

The absolute maximum is ___ occurring at $x = __________$

The absolute minimum is ___ occurring at $x = __________$
Question 10. (4 points each) Compute the following limits. Any uses of L’Hopital’s rule must be justified for full credit.

A. \( \lim_{x \to 2} \frac{x^3 - 8}{x^2 - 3x + 2} \)

B. \( \lim_{x \to \infty} \frac{x^2 + 3x + 2}{e^x} \)

C. \( \lim_{x \to 0} \frac{x^3}{x^3 - 3} \)

D. \( \lim_{t \to 0} \frac{e^t - t - \cos t}{t^2} \)

E. \( \lim_{x \to 0^+} (2x)^{3x} \)