Should get your exam back from your TA in lab by next Wed-ish.

MML 6 (due Mon) is like:

3. 1/ 4, 15, 19, 28, 37, 39, 47, 58, 61, 62
3.1 Intro to the Derivative

Back in 2.1:

\[ s = \text{position} \]

\[ \text{secant line} \]

\[ (\text{slope of line } s) = (\text{avg. vel. on } a \leq t \leq b) \]

\[ = \frac{s(b) - s(a)}{b - a} \]
The tangent line at \( t = a \) is given by:

\[
\text{Slope of line } T = \left( \text{instantaneous vel. at } t = a \right)
\]

\[
= \lim_{b \to a} \left( \frac{S(b) - S(a)}{b - a} \right)
\]

For a general function \( f(x) \):
\( y = f(x) \)

\[ \text{slope at } \xi = \frac{f(b) - f(a)}{b - a} \]

\( = \left\{ \text{average rate of change of } y = f(x) \right\} \]

\( \text{with respect to } x \), for \( a \leq x \leq b \).

\[ \text{slope of } T = \left\{ \text{instantaneous rate of change} \right\} \]

\( \text{of } f(x) \text{ w.r.t. } x \text{ at } x = a \)

\[ = \lim_{\xi \to a} \left( \frac{f(\xi) - f(a)}{\xi - a} \right). \]
If the derivative exists, then \( f \) is "differentiable" at \( x = a \).

Alternate notations:

\[
\text{slope } \frac{dT}{dt} = \lim_{h \to 0} \left( \frac{f(a+h) - f(a)}{h} \right) = \text{derivative of } f \text{ at } a
\]
Slope of $T$
\[= \lim_{h \to 0} \left( \frac{f(x+h)-f(x)}{h} \right) \]

= deriv. of $f$ at some input $x$.
For a given \( f(x) \), the function whose output at any \( x \) is the derivative of \( f \) at \( x \) is called \( f' \).

\[
f'(x) = \begin{cases} \text{slope of tangent line to graph} \\ \frac{df}{dx} \text{ at } x \end{cases}
\]

**Def:** \( f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \)

**Ex:** a) Find the derivative of \( f(x) = x^2 \) at \( x = 1 \).

b) Find the derivative of \( f(x) = x^4 \) at any input \( x \). (i.e. find \( f'(x) \)).
\textbf{a)}

\[
\frac{dy}{dx}
\]

(\text{slope of the tangent line at } x = 1)

\[
\lim_{x \to 1} \left( \frac{f(x) - f(1)}{x - 1} \right) = \lim_{x \to 1} \left( \frac{x^2 - 1}{x - 1} \right)
\]

\[
= \lim_{x \to 1} \left( \frac{(x-1)(x+1)}{x-1} \right) = \lim_{x \to 1} (x+1) = 2
\]

\textbf{b)}

\[
\begin{align*}
    f(x) &= x^2, \\
    f'(x) &= \lim_{h \to 0} \left( \frac{f(x+h) - f(x)}{h} \right) \\
    f(\square) &= (\square)^2
\end{align*}
\]
\[
\lim_{h \to 0} \left( \frac{(x+h)^2 - x^2}{h} \right) = \lim_{h \to 0} \left( \frac{x^2 + 2hx + h^2 - x^2}{h} \right) = \lim_{h \to 0} \left( \frac{h(2x+h)}{h} \right) = \lim_{h \to 0} (2x+h) = 2x.
\]

So for \( f(x) = x^2 \), \( f'(x) = 2x \).

Note: The equation of the tangent line to \( f(x) = x^2 \) at \( x = 1 \) is:

\[
y - 1 = 2(x - 1) \quad \text{or} \quad y = 2x - 1.
\]
3.1 19 \[ f(x) = x^2 - 4, \quad P(2,0) \]

\[ \lim_{x \to a} \left( \frac{f(x) - f(a)}{x - a} \right) = \lim_{x \to 2} \left( \frac{x^2 - 4 - 2^2 - 4}{x - 2} \right) = \lim_{x \to 2} \left( \frac{(x^2 - 4) - (2^2 - 4)}{x - 2} \right) \]

\[ = \lim_{x \to 2} \left( \frac{(x-2)(x+2)}{x-2} \right) = \lim_{x \to 2} (x+2) = 4 \]

b) Tan line eqn at \( P(2,0) \): \[ y - 0 = 4(x - 2) \]

or \[ y = 4x - 8. \]
3.1 (39) \( f(x) = 5x^2 - 6x + 1; \quad a = 2 \)

a) \( f'(x) = \lim_{h \to 0} \left( \frac{f(x+h) - f(x)}{h} \right) \)

\[
= \lim_{h \to 0} \left( \frac{(5(x+h)^2 - 6(x+h) + 1) - (5x^2 - 6x + 1)}{h} \right)
\]

\[
= \lim_{h \to 0} \left( \frac{5x^2 + 10xh + 5h^2 - 6x - 6h + 1 - 5x^2 + 6x - 1}{h} \right)
\]

\[
= \lim_{h \to 0} \left( \frac{10xh + 5h^2 - 6h}{h} \right) = \lim_{h \to 0} \frac{h(10x + 5h - 6)}{h}
\]

\[
= \lim_{h \to 0} (10x - 6) = f'(x).
\]