5.2 (8) Prove that \( \int_a^b c f(x) \, dx = c \int_a^b f(x) \, dx. \)

By definition, \( \int_a^b c f(x) \, dx = \lim_{n \to \infty} \left( \sum_{i=1}^n c f(x_i) \Delta x \right) \),

where \( \Delta x = \frac{b-a}{n}, \quad x_i = a + n \Delta x. \)

Since, for each \( n, \) \( \sum_{i=1}^n c f(x_i) \Delta x = c \sum_{i=1}^n f(x_i) \Delta x, \)

we now have (above) = \( \lim_{n \to \infty} \left( c \sum_{i=1}^n f(x_i) \Delta x \right) \).

Also, we know that \( \lim_{n \to \infty} \left( c \cdot g(n) \right) = c \cdot \lim_{n \to \infty} g(n), \) so

\( \text{(above)} = c \cdot \lim_{n \to \infty} \left( \sum_{i=1}^n f(x_i) \Delta x \right). \)
4.9 Today: Antiderivatives and the rules for.

Note: we can't say \( x^2 \) is \( \int \) the antiderivative of \( 2x \), because

\[
x^2 + 1, \quad x^2 - 10, \quad x^2 + e, \quad x^2 - 12164, \quad \ldots
\]

are all antiderivatives of \( 2x \).

We say the general antiderivative of \( 2x \) is \( x^2 + C \).

Notation: we know that \( \int_a^b f(x) \, dx \).
you are to find an antideriv of \( f(x) \) then plug in \( b \) and \( a \) and subtract. So we use

\[
\int f(x) \, dx
\]

as the symbol for the general antideriv. of \( f(x) \).

So we could write:

\[
\int 2x \, dx = x^2 + C.
\]

"general antideriv. of \\
2x with respect to \( x \)"

**Antiderivative Properties / Rules:**

- \( \int k f(x) \, dx = k \int f(x) \, dx \)
\[ \int (f(x) + g(x)) \, dx = \int f(x) \, dx + \int g(x) \, dx \]

Note: \[ \int f(x)g(x) \, dx \neq \int f(x) \, dx \cdot \int g(x) \, dx \]

\[ \int \frac{f(x)}{g(x)} \, dx \neq \frac{\int f(x) \, dx}{\int g(x) \, dx} \]

\[ \int x^p \, dx = \frac{1}{p+1} x^{p+1} + C \]

\[ \frac{d}{dx} (x^n) = nx^{n-1} \]

\[ \frac{d}{dx} \left( \frac{1}{n} x^n \right) = x^{n-1} \quad \frac{d}{dx} \left( \frac{1}{n+1} x^{n+1} \right) = x^n \]


\[ \int x^2 \, dx = \frac{1}{3} x^3 + C \]

\[ \int \frac{1}{x^4} \, dx = \int x^{-4} \, dx = -\frac{1}{3} x^{-3} + C \]

\[ = -\frac{1}{3x^3} + C \]

\[ \frac{3}{2} + 1 = \frac{5}{2} \]

\[ \int 4 \sqrt{x} \, dx = \int 4 \cdot x^{\frac{1}{2}} \, dx = 4 \cdot \frac{2}{3} x^{\frac{3}{2}} + C \]

\[ = \frac{8}{3} x^{\frac{3}{2}} + C \]

\[ 4 \left( \frac{2}{3} x^{\frac{3}{2}} + C \right) \]

\[ = \frac{8}{3} \left( \sqrt{x} \right)^{\frac{3}{2}} + C \]

\[ 4 \left( \frac{3}{3} x^{\frac{3}{2}} + 4C \right) \]
Can rewrite all other rules:

\[
\frac{d}{dx}(e^x) = e^x, \quad \text{so} \quad \int e^x \, dx = e^x + C
\]

\[
\frac{d}{dx}(\ln|x|) = \frac{1}{x}, \quad \text{so} \quad \int \frac{1}{x} \, dx = \ln|x| + C
\]

\[
\frac{d}{dx}(\sin(x)) = \cos(x), \quad \text{so} \quad \int \cos(x) \, dx = \sin(x) + C
\]

\[
e^{1/2}
\]

\[
\frac{d}{dx}(\sin(ax)) = a \cdot \cos(ax), \quad \text{so} \quad \frac{d}{dx}(\frac{1}{a} \cdot \sin(ax)) = \cos(ax),
\]

\[
\text{so} \quad \int \cos(ax) \, dx = \frac{1}{a} \sin(ax) + C
\]
Similarly, \( \int e^{5x} \, dx = \frac{1}{5} e^{5x} + C \)

Note: \( \int e^{5x^2} \, dx \neq \frac{1}{5} e^{5x^2} + C \), since
\[
\frac{d}{dx} \left( \frac{1}{5} e^{5x^2} + C \right) = \frac{1}{5} e^{5x^2} \cdot 10x + 0 = \frac{1}{5} e^{5x^2} \cdot 10x + C
\]

4.9 79. Find \( \int (\sec^2(x) - 1) \, dx = \tan(x) - x + C \)

43. \( \int (3t^2 + \sec^2(2t)) \, dt = t^3 + \frac{1}{2} \tan(2t) + C \)
An "initial value problem" is a problem where you're told what $f'(x)$ is, and what $f$ (some number) is, and asked to find the formula for $f(x)$.

**Ex:** 4.9 (67) $f'(x) = 2x - 3$, $f(0) = 4$. 
Solution: \( f'(x) = 2x - 3 \Rightarrow f(x) = \int (2x - 3) \, dx \)

\[ = x^2 - 3x + C \]

\( f(0) = 4 \Rightarrow \) when \( x = 0 \), output must = 4, so

\[ 0^2 - 3(0) + C = 4 \Rightarrow C = 4. \]

So \( f(x) = x^2 - 3x + 4 \).