MML 14 is like: (due Thurs 10/22)

5.1/ 11, 17, 29, 41,
5.2/ 7, 16, 21, 23, 26, 33, 35

Should get your tests back Tues/Wed in lab.
Question: use linearization to approximate \( \frac{1}{5.02} \).

Solution:

So, define \( f(x) = \frac{1}{x} \).

The linearization of this \( f \) at \( x = 5 \) is:

\[
\begin{align*}
f(5) &= \frac{1}{5} = 0.2, \\
f'(x) &= -\frac{1}{x^2}, \\
f'(5) &= -\frac{1}{25} = -0.04.
\end{align*}
\]

So \( L(x) = 0.2 - 0.04(x - 5) \).
So \( f(5.02) = \frac{1}{5.02} \approx L(5.02) = 0.2 - 0.04(5.02 - 5) \)

\[
= 0.2 - 0.04(0.02) = 0.2 - 0.0008 = 0.1992
\]

0P: \( f(5.02) \approx f(5) + dy \) when \( x \) goes from \( 5 \) to \( 5.02 \)

\[
= f(5) + f'(5) \cdot \Delta x
\]

\[
= \frac{1}{5} + \frac{-1}{25} (0.02) = \text{same answer.}
\]
4.8: one solution to \( f(x) = 0 \) is this \( x \)-value.

A computer could use the following scheme to home-in on the noted solution:
$L(x) = f(x_0) + f'(x_0)(x-x_0)$

$y = f(x)$

$x_1$ is the $x$-value where $L(x) = 0$.

So:

$f(x_0) + f'(x_0)(x-x_0) = 0$

$f'(x_0)(x-x_0) = -f(x_0)$

$x-x_0 = \frac{-f(x_0)}{f'(x_0)}$

$x_1 = x = x_0 - \frac{f(x_0)}{f'(x_0)}$

Initial guess.
This process is repeated, so:

\[ x_0 = \text{initial guess for sol. to } f(x) = 0, \]

\[ x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad \text{for } n=0,1,2,\ldots \]

"Newton's Method"

Note: If \( x_0 \) is not good enough, then Newton's Method might diverge.

\( \text{The } x_n \)'s might have no limit.\)
\[ x_i = x_0 - \frac{f(x_0)}{f'(x_0)} \rightarrow 0 \]
4.8 (6) \[ f(x) = x^2 - 2x - 3; \ x_0 = 2. \]

Use Newton's method to go after a solution to

\[ f(x) = x^2 - 2x - 3 = 0, \text{ starting with } x_0 = 2. \]

\[ x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \]

\[ f'(x) = 2x - 2 \]

\[ x_0 = 2, \]

\[ x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2 - \frac{f(2)}{f'(2)} = 2 - \frac{(2)^2 - 2(2) - 3}{2(2) - 2} \]

\[ = 2 - \frac{-3}{2} = \frac{7}{2} = 3.5. \]
\[ x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = \frac{7}{2} - \frac{f\left(\frac{7}{2}\right)}{f'\left(\frac{7}{2}\right)} \]

\[ = \frac{7}{2} - \frac{\left(\frac{7}{2}\right)^2 - 2 \cdot \left(\frac{7}{2}\right) - 3}{2 \cdot \left(\frac{7}{2}\right) - 2} = \frac{7}{2} - \frac{\frac{9}{4}}{5} \]

\[ = \frac{7}{2} - \frac{9}{20} = \frac{61}{20} = 3.05 \]
Area under $y = f(x)$ from $x = a$ to $x = b$ is approximately the sum of the $n$ areas, which is:
\[ f(x_1^*) \Delta x + f(x_2^*) \Delta x + f(x_3^*) \Delta x + f(x_4^*) \Delta x \]

A regular Riemann sum with 4 subintervals.

Could do:

Regular right Riemann sum.

Can also do left sums and middle sums.
Note: if we use more-and-more, smaller-and-smaller rectangles, our approximation will get closer-and-closer to the exact area:

The generic, right, regular Riemann sum would be:
\[ y = f(x) \]

Use \( n \) subintervals.

\[ \Delta x = \frac{b-a}{n} \]

\[ f(x_1) \Delta x + f(x_2) \Delta x + \ldots + f(x_n) \Delta x. \]

So, should be true that

\[
\left[ \text{exact area under} \right] f, \text{ from } a \text{ to } b = \lim_{n \to \infty} \left( f(x_1) \Delta x + \ldots + f(x_n) \Delta x \right).
\]

more next time.