Math 171 Fall 2015
More Exam 3 Practice

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Last Name: __________________________
First Name: _________________________
Student ID: _________________________
Lab Section: ________________________

Remember to show all of your work and provide all necessary explanations for full credit. Good Luck!
Problem 1. True or False. No justifications are necessary.

(a) The function \( f(x) = |x| \) is integrable on \([-2, 2]\).

(b) If \( f \) is integrable on \([a, b]\) then \( \int_a^b f(x) \, dx = \int_b^a f(x) \, dx \).

(c) The average value of \( f(x) = x \) on \( 0 \leq x \leq 10 \) is 5.

(d) \( \int_2^5 \sin(x) \, dx = \lim_{n \to \infty} \left( \frac{3}{n} \sum_{k=1}^{n} \left( \sin \left( 2 + k \frac{3}{n} \right) \right) \right) \).

(e) If \( g(x) = \int_3^x \tan(t^2) \, dt \), then \( g'(x) = \tan(x^2) - \tan(3^2) \).

(f) If \( f \) is continuous on \([a, b]\) and the average value of \( f(x) \) on \([a, b]\) is zero, then \( f \) must have an \( x \)-intercept in \((a, b)\).

(g) If \( \int_2^7 f(x) \, dx = 5 \) and \( \int_5^7 f(x) \, dx = 8 \), then \( \int_2^5 f(x) \, dx = 13 \).

(h) \( \sum_{k=3}^{50} 10 = 470 \).
Problem 2. Use the graph of $y = f(x)$ shown here to answer the questions below.

(a) Find the value of $\int_0^8 f(x) \, dx$

(b) Find the average value of $f$ on $[0, 8]$

(c) Find the value of $\int_3^0 f(x) \, dx$

(d) Find the value of $\int_3^6 f(x) \, dx + \int_6^8 f(x) \, dx$

(d) If $A(x) = \int_1^x f(t) \, dt$,
   (i) the value of $A'(4)$

   (ii) all $x$ values in $(0, 8)$ where $A'(x) = 0$

   (iii) the maximum output value achieved by $A(x)$ for $0 \leq x \leq 8$

(e) In the picture above, draw the graph of $A(x) = \int_1^x f(t) \, dt$ as accurately as you can.
Problem 3. Evaluate the following integrals. For full credit, $+C$ must be included in the answer for any indefinite integral.

(a) \[ \int \left( \frac{2}{x} - \frac{4}{x^3} - 3\sqrt{x} \right) \, dx \]

(b) \[ \int_0^\pi \left( 5 + 3 \sin(x/2) \right) \, dx \]

(c) \[ \int \left( 2x^4 - \frac{3}{x^2 + 1} \right) \, dx \]

(d) \[ \int_0^1 e^{3x} \, dx \]
Problem 4. Evaluate the following integrals using $u$-substitutions. Again, the solution to any indefinite integral must include $+C$ for full credit.

(a) $\int (3x^2 + 5) \cos(x^3 + 5x + 1) \, dx$

(b) $\int_0^1 \frac{x^2}{4 + 2x^3} \, dx$

(c) $\int x^4(2x^5 + 7)^8 \, dx$

(d) $\int \frac{x^2}{\sqrt{1-x^6}} \, dx \quad$ Hint: use $u = x^3$. 
Problem 5.

Known formulas: \[ \sum_{k=1}^{n} k = \frac{n(n + 1)}{2}, \quad \sum_{k=1}^{n} k^2 = \frac{n(n + 1)(2n + 1)}{6}. \]

(a) Evaluate \[ \sum_{k=2}^{50} (3k + k^2 - 2) \]

(b) Evaluate the telescoping sum \[ \sum_{k=1}^{100} \left( \frac{1}{k} - \frac{1}{k + 2} \right). \]
Problem 6. For the function \( f(x) = \frac{1}{\sqrt{x}} \), do the following.

(a) Find \( \bar{f} \), the average value of \( f \) on the interval \( 1 \leq x \leq 4 \).

(b) Find all values \( c \) in \( (1, 4) \) such that \( f(c) = \bar{f} \).

Problem 7. Suppose an object moves in a straight line with velocity \( v(t) = 2t^2 + 5 \), and at time \( t = 0 \), the object’s position is \( s(0) = 2 \). Find the object’s acceleration function \( a(t) \) and its position function \( s(t) \).