55. \[ I = \iint_R (y - 2x^2) \, dA \]

\[ = \int_{R_1} (y - 2x^2) \, dA + \int_{R_2} (y - 2x^2) \, dA \]

Can use vertical cross sections to write each integral.

\[ I = \int_{-1}^{0} \int_{-x-1}^{x+1} (y - 2x^2) \, dy \, dx + \int_{0}^{1} \int_{x-1}^{1-x} (y - 2x^2) \, dy \, dx \]

\[ = \int_{-1}^{0} \left( \frac{1}{2} y^2 - 2x^2 y \right)_{y=-x-1}^{y=x+1} \, dx + \int_{0}^{1} \left( \frac{1}{2} y^2 - 2x^2 y \right)_{y=x-1}^{y=1-x} \, dx \]

\[ = \int_{-1}^{0} \left( \frac{1}{2} [(x+1)^2 - (-x+1)^2] - 2x^2 \left[ (x+1) - (x-1) \right] \right) \, dx \]

\[ + \int_{0}^{1} \left( \frac{1}{2} [(1-x)^2 - (-1-x)^2] - 2x^2 \left[ (1-x) - (-1-x) \right] \right) \, dx \]

\[ = \int_{-1}^{0} \left( -4 (x^3 + x^2) \right) \, dx + \int_{0}^{1} \left( -4 (x^2 - x^3) \right) \, dx \]

\[ = \left[ -x^4 - \frac{4}{3} x^3 \right]_{-1}^{0} + \left[ \frac{4}{3} x^3 + x^4 \right]_{0}^{1} = -\left( -1 \right)^4 - \frac{4}{3} (1)^3 + \left( \frac{4}{3} + 1 \right)^4 \]
\[ = 1 - \frac{4}{3} - \frac{4}{3} + 1 = \frac{-2}{3}. \]

We might not want to use horizontal cross sections after splitting \( R \) into \( R_1 \) and \( R_2 \) as we did.

But instead, we could have split \( R \) horizontally into \( R_3 \) and \( R_4 \), say, and then used horizontal cross sections.

Just as we could choose which variable to differentiate first w.r.t. in a second derivative, e.g., \( \frac{\partial^2 f}{\partial x \partial y} \), we could choose which variable to integrate w.r.t. in a double integral, so that the computation becomes easier.
Properties of Double Integrals

Let $f(x,y)$ and $g(x,y)$ be continuous functions over region $R$.

1. $\iint_R cf(x,y)\,dA = c\iint_R f(x,y)\,dA$ (constant multiple)

2. Sum/difference

   $\iint_R (f(x,y) + g(x,y))\,dA = \iint_R f(x,y)\,dA + \iint_R g(x,y)\,dA$

3. Domination. If $f(x,y) \geq g(x,y)$ on $R$, then

   $\iint_R f(x,y)\,dA \geq \iint_R g(x,y)\,dA$

4. Additivity

   $\iint_R f(x,y)\,dA = \iint_{R_1} f(x,y)\,dA + \iint_{R_2} f(x,y)\,dA$ where $R$ is the union of nonoverlapping regions $R_1$ and $R_2$. 

\[ R \rightarrow R_1 \cup R_2 \]
57. Find the volume of the region bounded above by the paraboloid \( z = x^2 + y^2 \) and below by the triangle enclosed by the lines \( y = x \), \( x = 0 \), and \( x + y = 2 \) in the \( x-y \) plane.

\[
V = \int_0^2 \int_0^1 (x^2 + y^2) \; dy \; dx = \int_0^1 \left( \frac{y^3}{3} \right) \left|_0^2 \right. \; dx
\]

\[
= \int_0^1 \left( \frac{x^2 (2-x) - x^3}{2} \right) \; dx
\]

\[
= \int_0^1 \left( \frac{8x - 12x^2 + 6x^2 - 2x^3}{2} \right) \; dx
\]

\[
= \int_0^1 \left( \frac{-2x^3 + 8x - 10x^2}{3} \right) \; dx
\]

\[
= \frac{1}{3} \left[ \frac{-2x^4 + 8x^2 - 10x^3}{3} \right] \left|_0^1 \right. = \frac{1}{3} (8 - 6 + 4 - 2) = \frac{4}{3}.
\]

\[0 \leq x \leq 1, \; 0 \leq y \leq 2 \text{ here, giving} \]

\[0 \leq z \leq 5.\]

This is a more accurate figure than the one drawn by hand above \( \odot \).