HW #3
Due Monday 9/21 by 4pm

Show your work in the blank area and put the final answer in the box.

1. (Ex 3.82) Consider writing onto a computer disk and then sending it through a certifier that counts the number of missing pulses. Suppose this number X has a Poisson distribution with parameter $\lambda=0.2$.
   A. What is the probability that a disk has exactly 1 missing pulse?
      
      $$P(X=1) = \frac{e^{-0.2}(0.2)^1}{1!} = 0.1637$$

      
      B. What is the probability that a disk has at least 2 missing pulses?
      
      $$P(X \geq 2) = 1 - P(X \leq 1)$$
      $$= 1 - P(X=0) - P(X=1)$$
      $$= 1 - \frac{e^{-0.2}(0.2)^0}{0!} - 0.1637 = 0.0176$$

   C. If two disks are independently selected, what is the probability that neither contains a missing pulse?
      
      $$P(\text{neither}) = P(X=0) \cdot P(X=0)$$
      $$= 0.81872 \times 0.81872 = 0.6703$$

2. (Ex 3.88) In proof testing of circuit boards, the probability that any particular diode will fail is 0.01. Suppose a circuit board contains 200 diodes.
   For parts B-D, give your answer to 4 decimal places.
   A. How many diodes in average would you expect to fail and what is the standard deviation of the number that fail? (3 pts)
      
      $$X \sim Bin(n=200, \ p=0.01)$$
      $$E(X) = np = 2$$
      $$\sigma^2 = np(1-p) = (200)(0.01)(0.99) = 1.9791$$

   B. What is the probability that at least four diodes will fail on a randomly selected board?
      
      $$P(X \geq 4) = 1 - P(X \leq 3) = 1 - P(X=0) - P(X=1) - P(X=2) - P(X=3)$$
      $$= 1 - \binom{200}{0}0.01^0(1-0.01)^{200} - \binom{200}{1}0.01^1(0.99)^{199} - \binom{200}{2}0.01^2(0.99)^{198}$$
      $$- \binom{200}{3}0.01^3(0.99)^{197}$$
      $$= 0.1420$$

   C. Based on the Poisson approximation, what is the probability that at least four diodes will fail on a randomly selected board?
      
      $$P(X \geq 4) = 1 - P(X \leq 3), \ \ \ \ \lambda = np = 2$$
      $$= 1 - \frac{e^{-2.20}}{0!} - \frac{e^{-2.21}}{1!} - \frac{e^{-2.22}}{2!} - \frac{e^{-2.23}}{3!}$$
      $$= 0.1427$$
Question 2 (circuit boards) continued...

D. If 5 boards are shipped to a particular customer, how likely is it that at least four of them will work properly? (A board works properly only if all of its diodes work.)

\[ P(\text{a board works properly}) = P(\text{all 200 diodes work}) = 0.1340 \]

\[ P(Y \geq 4) = 1 - P(Y \leq 3) = 1 - 0.0014 = 0.9986 \]

3. (4.2) Suppose the reaction temperature \( X \) (in °C) in a certain chemical process has a uniform distribution with \( A = -5 \) and \( B = 5 \).

A. Sketch the pdf below:

\[ f(x) = \frac{1}{B-A} = \frac{1}{10} = 0.1, \quad -5 \leq x \leq 5 \]

B. Compute \( P(X < 0) \).

\[ P(X < 0) = \int_{-5}^{0} \frac{1}{10} \, dx = 5 \cdot 0.1 = 0.5 \]

C. Compute \( P(-1 \leq X \leq 2) \).

\[ P(-1 \leq X \leq 2) = \int_{-1}^{2} \frac{1}{10} \, dx = 3 \cdot 0.1 = 0.3 \]

Note: If \( Y \sim \text{unif}(A,B) \), then \( E(Y) = \frac{A+B}{2} \) and \( V(Y) = \frac{(B-A)^2}{12} \).

D. Find \( E(X) \) and \( V(X) \).

\[ E(X) = \frac{A+B}{2} = 0 \]

\[ V(X) = \frac{(B-A)^2}{12} = \frac{100}{12} = 8.333 \]

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