Questions 1 through 8: A random sample of 20 concrete cylinders was obtained and their strength was measured. The ordered data is shown below along with some summary statistics:

<table>
<thead>
<tr>
<th>5.8</th>
<th>6.1</th>
<th>6.6</th>
<th>7</th>
<th>7.1</th>
<th>7.2</th>
<th>7.4</th>
<th>7.8</th>
<th>7.8</th>
<th>8.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.3</td>
<td>8.3</td>
<td>8.5</td>
<td>8.9</td>
<td>9.2</td>
<td>9.7</td>
<td>9.8</td>
<td>11.2</td>
<td>12.6</td>
<td>14.1</td>
</tr>
</tbody>
</table>

\[
\sum x_i = 171.5 \quad \sum x_i^2 = 1555 \quad \sum (x_i - \bar{x})^2 = 84.1175
\]

1. What is the mean cylinder strength?

\[
\bar{x} = \frac{\sum x_i}{n} = \frac{171.5}{20} = 8.575
\]

2. What is the variance of the cylinder strengths?

\[
S^2 = \frac{\sum (x_i - \bar{x})^2}{n-1} = \frac{84.1175}{19} = 4.42723
\]

3. What is the standard deviation of the cylinder strengths?

\[
S = \sqrt{S^2} = 2.1041
\]

4. What is the median cylinder strength?

\[
\text{Median} = \frac{8.1 + 8.3}{2} = 8.2
\]

5. What is the value of the first quartile?

\[
Q_1 = \frac{7.1 + 7.2}{2} = 7.15
\]

6. What is the value of the third quartile?

\[
Q_3 = \frac{9.2 + 9.7}{2} = 9.45
\]

7. What is the interquartile range?

\[
IQR = Q_3 - Q_1 = 9.45 - 7.15 = 2.3
\]
8. Using the boxplot convention, are there any outliers in the data? If so, what are the values?

\[ Q_1 - 1.5 \times IQR = 7.15 - 3.45 = 3.7 \]
\[ Q_3 + 1.5 \times IQR = 9.45 + 3.45 = 12.9 \]

1 outlier = 14.1

Questions 9 through 11: DNA molecules consist of chemically linked sequences of the bases adenine (A), cytosine (C), guanine (G) and thymine (T). An (ordered) sequence of three bases is called a codon. A based may appear multiple times in a codon.

9. How many different codons are there?

\[ 4 \times 4 \times 4 = 64 \]

10. How many codons consist of three different bases?

\[ P_{3,4} = 4 \times 3 \times 2 = 24 \]

11. The bases A and G are purines, while C and T are pyrimidines. How many codons are there whose 1\(^{st}\) and 3\(^{rd}\) bases are purines and whose 2\(^{nd}\) base is a pyrimidine?

Purine/ Pyrimidine/ Purine

\[ 2 \times 2 \times 2 = 8 \]

Questions 12 through 15: A sample of five items is drawn from a large lot in which 10% of the items are defective. Let X represent the number of defective items observed in a random sample of five items. The following table gives the probabilities associated with each possible value of X.

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>P(X=x)</td>
<td>?</td>
<td>?</td>
<td>0.0729</td>
<td>0.0081</td>
<td>0.00045</td>
<td>0.00001</td>
</tr>
</tbody>
</table>

12. Find the P(X=1).

\[ P(X=1) = \binom{5}{1} \times 0.1^1 \times 0.9^4 = 0.32805 \]

13. Find the P(X≤2).

\[ P(X\leq2) = 1 - P(X=5) - P(X=4) - P(X=3) \]
\[ = 1 - 0.00001 - 0.00045 - 0.00008 \]
\[ = 0.99947 \]

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= 0.99135
14. Find \( E(X) \).
\[
E(X) = np = 5 \times 0.1 = 0.5
\]

15. Find \( V(X) \).
\[
V(X) = np(1-p) = 5 \times 0.1 \times 0.9 = 0.45
\]

**Questions 16 through 19:** Let \( X \) be the number of messages received by a computer bulletin board in one hour. It is known that \( X \) follows the Poisson distribution with a mean rate of 8 messages per hour.

16. Find \( E(X) \).
\[
E(X) = \lambda = 8
\]

17. Find \( V(X) \).
\[
V(X) = \lambda = 8
\]

18. What is the probability that 5 messages are received in a given hour.
\[
P(X = 5) = \frac{e^{-8} \times 8^5}{5!} = 0.091604
\]

19. What is the probability that 10 messages are received in a 2 hour span.
\[
Y \sim \text{Poisson}(\lambda = 2 \times 8)
\]
\[
P(Y = 10) = \frac{e^{-16} \times 16^{10}}{10!} = 0.034977
\]

**Questions 20 through 22:** Let \( E \) be the event that a new car requires engine work under warranty and let \( T \) be the event that the car requires transmission work under warrant. Suppose that \( P(E) = 0.10 \), \( P(T) = 0.02 \) and \( P(E \cap T) = 0.01 \).

20. What is the probability that the car needs work on either the engine, the transmission or both?
\[
P(E \cup T) = P(E) + P(T) - P(E \cap T)
\]
\[
v = 0.1 + 0.02 - 0.01 = 0.11
\]

\[ P(E \cap T) = 0.01 \]

\[ P(E) \times P(T) = 0.1 \times 0.02 = 0.002 \]

\[ P(E \cap T) \neq P(E) \times P(T) \quad \text{so E and T are not independent} \]

22. The random variables given below are either discrete or continuous. Which random variable is discrete?

- A = length of a concrete cylinder
- B = current temperature at Pullman
- C = number of telephone calls received in a day
- D = weight of a car engine

22. C

**Questions 23 through 25:** Of cars sold in a certain state, 45% have small engines, 35% have medium engines and 20% have the large engines. Of cars with small engines, 10% fail an emissions test (within 2 years of purchase), 12% of cars with medium engines fail and 15% of those with large engines fail.

23. For cars with medium size engines, what is the probability that a randomly selected car will pass the emissions test?

\[ 1 - 0.12 = 0.88 \]

24. What is the probability that a randomly selected car will fail the emissions test?

\[ P(F) = P(F \cap S) + P(F \cap M) + P(F \cap L) \]
\[ = 0.45 \times 0.10 + 0.35 \times 0.12 + 0.20 \times 0.15 \]
\[ = 0.117 \]

25. Given that a car has failed the emissions test, what is the probability that is has a small engine?

\[ P(S | F) = \frac{P(S \cap F)}{P(F)} = \frac{0.045}{0.117} = 0.3846 \]