5.1 **EXERCISES**

1. Is \( \lambda = 2 \) an eigenvalue of \[
\begin{pmatrix}
3 & 2 \\
3 & 8
\end{pmatrix}
\] Why or why not?

2. Is \( \lambda = -2 \) an eigenvalue of \[
\begin{pmatrix}
7 & 3 \\
3 & -1
\end{pmatrix}
\] Why or why not?

3. Is \[
\begin{pmatrix}
1 \\
4
\end{pmatrix}
\] an eigenvector of \[
\begin{pmatrix}
-3 & 1 \\
-3 & 8
\end{pmatrix}
\] If so, find the eigenvalue.

4. Is \[
\begin{pmatrix}
-1 + \sqrt{2} \\
1
\end{pmatrix}
\] an eigenvector of \[
\begin{pmatrix}
2 & 1 \\
1 & 4
\end{pmatrix}
\] If so, find the eigenvalue.

5. Is \[
\begin{pmatrix}
-3 \\
1
\end{pmatrix}
\] an eigenvector of \[
\begin{pmatrix}
3 & 7 & 9 \\
-4 & -5 & 1 \\
2 & 4 & 4
\end{pmatrix}
\] If so, find the eigenvalue.

6. Is \[
\begin{pmatrix}
1 \\
-2
\end{pmatrix}
\] an eigenvector of \[
\begin{pmatrix}
3 & 6 & 7 \\
3 & 3 & 7 \\
5 & 6 & 5
\end{pmatrix}
\] If so, find the eigenvalue.

7. Is \( \lambda = 4 \) an eigenvalue of \[
\begin{pmatrix}
3 & 0 & -1 \\
2 & 3 & 1 \\
-3 & 4 & 5
\end{pmatrix}
\] If so, find one corresponding eigenvector.

8. Is \( \lambda = 3 \) an eigenvalue of \[
\begin{pmatrix}
1 & 2 & 2 \\
3 & -2 & 1 \\
0 & 1 & 1
\end{pmatrix}
\] If so, find one corresponding eigenvector.

In Exercises 9–16, find a basis for the eigenspace corresponding to each listed eigenvalue.

9. \( A = \begin{pmatrix} 5 & 0 \\ 2 & 1 \end{pmatrix} \), \( \lambda = 1, 5 \)

10. \( A = \begin{pmatrix} 10 & -9 \\ 4 & -2 \end{pmatrix} \), \( \lambda = 4 \)

11. \( A = \begin{pmatrix} 4 & -2 \\ -3 & 9 \end{pmatrix} \), \( \lambda = 10 \)

12. \( A = \begin{pmatrix} 7 & 4 \\ -3 & -1 \end{pmatrix} \), \( \lambda = 1, 5 \)

13. \( A = \begin{pmatrix} 4 & 0 & 1 \\ -2 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix} \), \( \lambda = 1, 2, 3 \)

14. \( A = \begin{pmatrix} 1 & 0 & -1 \\ 1 & -3 & 0 \\ 4 & -13 & 1 \end{pmatrix} \), \( \lambda = -2 \)

15. \( A = \begin{pmatrix} 4 & 2 & 3 \\ -1 & 1 & -3 \\ 2 & 4 & 9 \end{pmatrix} \), \( \lambda = 3 \)

16. \( A = \begin{pmatrix} 3 & 0 & 2 \\ 1 & 3 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 4 \end{pmatrix} \), \( \lambda = 4 \)

Find the eigenvalues of the matrices in Exercises 17 and 18.

17. \( A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 2 & 5 \\ 0 & 0 & -1 \end{pmatrix} \)

18. \( A = \begin{pmatrix} 4 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & -3 \end{pmatrix} \)

19. For \( A = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} \), find one eigenvalue, with no calculation. Justify your answer.

20. Without calculation, find one eigenvalue and two linearly independent eigenvectors of \( A = \begin{pmatrix} 5 & 5 & 5 \\ 5 & 5 & 5 \\ 5 & 5 & 5 \end{pmatrix} \). Justify your answer.
36. Consider an mxn matrix A with the property that the cofactor of each entry is a power of 2. Find an m x n matrix B with the property that the cofactor of each entry is twice the power of 2.

37. Consider an m x n matrix A with an entry of 2 in the (i, j) position. Show that there is an mxn matrix B with the property that the cofactor of each entry is twice the power of 2.

38. Let A be an mxn matrix with the property that the cofactor of each entry is a power of 2. Find an mxn matrix B with the property that the cofactor of each entry is twice the power of 2.

39. Consider an mxn matrix A with the property that the cofactor of each entry is a power of 2. Find an mxn matrix B with the property that the cofactor of each entry is twice the power of 2.

40. Consider an mxn matrix A with the property that the cofactor of each entry is a power of 2. Find an mxn matrix B with the property that the cofactor of each entry is twice the power of 2.

41. Consider an mxn matrix A with the property that the cofactor of each entry is a power of 2. Find an mxn matrix B with the property that the cofactor of each entry is twice the power of 2.
According to their multiplications, the matrices in Exercises 15–17, the characteristic equations are:

\[
\begin{bmatrix}
-2 & 3 \\
1 & -2
\end{bmatrix}
\quad \begin{bmatrix}
2 & 0 \\
1 & 1
\end{bmatrix}
\quad \begin{bmatrix}
-2 & 0 \\
1 & 2
\end{bmatrix}
\]

Exercises 9–14 require recitations from Section 3.1. Find the expansion of the special formula for \( A \times A \) determinants. According to the characteristic polynomial of each matrix, where either a diagonal or a non-diagonal is involved, the algebraic multiplicity of \( \lambda \) is equal to the geometric multiplicity of \( \lambda \).

Find the characteristic polynomial and the eigenvectors of the matrices:

\[
\begin{bmatrix}
2 & 0 \\
1 & 1
\end{bmatrix}
\quad \begin{bmatrix}
-2 & 0 \\
1 & 2
\end{bmatrix}
\quad \begin{bmatrix}
-2 & 3 \\
1 & -2
\end{bmatrix}
\]

These in Exercises 1–8.

Find the characteristic equation and eigenvectors of \( A = \begin{bmatrix} 2 & 4 \\ -4 & 1 \end{bmatrix} \).

**Practice Problem**

- **Numerical Notes**

- **The Characteristic Equation**

- **5.2 Exercises**

- **5.4, 6.1, 5.2, 1.2, 2.3**

- **9.4, 10.5, 11.2, 3.2**

- **9.4, 10.5, 11.2, 3.2**
15. \[
\begin{bmatrix}
4 & -7 & 0 & 2 \\
0 & 3 & -4 & 6 \\
0 & 0 & 3 & -8 \\
0 & 0 & 0 & 1
\end{bmatrix}
\quad 16. \begin{bmatrix}
5 & 0 & 0 & 0 \\
8 & -4 & 0 & 0 \\
0 & 7 & 1 & 0 \\
1 & -5 & 2 & 1
\end{bmatrix}
\]
17. \[
\begin{bmatrix}
3 & 0 & 0 & 0 \\
-5 & 1 & 0 & 0 \\
3 & 8 & 0 & 0 \\
0 & -7 & 2 & 1 \\
-4 & 1 & 9 & -2
\end{bmatrix}
\]
18. It can be shown that the algebraic multiplicity of an eigenvalue \(\lambda\) is always greater than or equal to the dimension of the eigenspace corresponding to \(\lambda\). Find \(A\) in the matrix \(A\) below such that the eigenspace for \(\lambda = 5\) is two-dimensional:
\[
A = \begin{bmatrix}
5 & -2 & 6 & -1 \\
0 & 3 & k & 0 \\
0 & 0 & 5 & 4 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]
19. Let \(A\) be an \(n \times n\) matrix, and suppose \(A\) has \(n\) real eigenvalues, \(\lambda_1, \ldots, \lambda_n\), repeated according to multiplicities, so that
\[
det(A - \lambda I) = (\lambda_1 - \lambda)(\lambda_2 - \lambda) \cdots (\lambda_n - \lambda)
\]
Explain why \(\det(A)\) is the product of the \(n\) eigenvalues of \(A\). (This result is true for any square matrix when complex eigenvalues are considered.)

20. Use a property of determinants to show that \(A\) and \(A^T\) have the same characteristic polynomial.

In Exercises 21 and 22, \(A\) and \(B\) are \(n \times n\) matrices. Mark each statement True or False. Justify each answer.

21. a. The determinant of \(A\) is the product of the diagonal entries in \(A\).
   b. An elementary row operation on \(A\) does not change the determinant.
   c. \((\det A)(\det B) = \det AB\)
   d. If \(\lambda + 5\) is a factor of the characteristic polynomial of \(A\), then 5 is an eigenvalue of \(A\).

22. a. If \(A\) is \(3 \times 3\), with columns \(a_1, a_2, a_3\), then \(\det A\) equals the volume of the parallelepiped determined by \(a_1, a_2, a_3\).
   b. \(\det A^T = (-1)^r \det A\).
   c. The multiplicity of a root \(r\) of the characteristic equation of \(A\) is called the algebraic multiplicity of \(r\) as an eigenvalue of \(A\).
   d. A row replacement operation on \(A\) does not change the eigenvalues.

A widely used method for estimating eigenvalues of a general matrix \(A\) is the \(QR\) algorithm. Under suitable conditions, this algorithm produces a sequence of matrices, all similar to \(A\), that become almost upper triangular, with diagonal entries that approach the eigenvalues of \(A\). The main idea is to factor \(A\) (or another matrix similar to \(A\)) in the form \(A = QR\), where \(Q^T = Q^{-1}\) and \(R\) is upper triangular. The factors are interchanged to form \(A_1 = R_1 Q_1\), which is again factored as \(A_1 = Q_2 R_2\); then to form \(A_2 = R_2 Q_2\), and so on. The similarity of \(A, A_1, \ldots\) follows from the more general result in Exercise 23.

23. Show that if \(A = QR\) with \(Q\) invertible, then \(A\) is similar to \(A_1 = RQ\).

24. Show that if \(A\) and \(B\) are similar, then \(\det A = \det B\).

25. Let \(A = \begin{bmatrix}
.6 & .3 \\
.4 & .7
\end{bmatrix}, \quad v_1 = \begin{bmatrix} 3/7 \\
4/7
\end{bmatrix}, \quad x_0 = \begin{bmatrix} .5 \\
.5
\end{bmatrix} \quad [Note: A is the stochastic matrix studied in Example 5 of Section 4.9.]
   a. Find a basis for \(\mathbb{R}^2\) consisting of \(v_1\) and another eigenvector \(v_2\) of \(A\).
   b. Verify that \(x_0\) may be written in the form \(x_0 = v_1 + cv_2\).
   c. For \(k = 1, 2, \ldots\), define \(x_k = A^k x_0\). Compute \(x_1\) and \(x_2\), and write a formula for \(x_k\). Then show that \(x_k \to v_1\) as \(k\) increases.

26. Let \(A = \begin{bmatrix} a & b \\
c & d \end{bmatrix}\) Use formula (1) for a determinant (given before Example 2) to show that \(\det A = ad - bc\). Consider two cases: \(a \neq 0\) and \(a = 0\).

27. Let \(A = \begin{bmatrix}
.5 & .2 & .3 \\
.3 & .8 & .3 \\
.2 & 0 & .4
\end{bmatrix}, \quad v_1 = \begin{bmatrix} .3 \\
.1 \\
-3
\end{bmatrix}, \quad v_2 = \begin{bmatrix} 1 \\
2 \\
1
\end{bmatrix}, \quad v_3 = \begin{bmatrix} -1 \\
0 \\
1
\end{bmatrix}, \quad w = \begin{bmatrix} 1 \\
1 \\
1
\end{bmatrix}
   a. Show that \(v_1, v_2, v_3\) are eigenvectors of \(A\). [Note: \(A\) is the stochastic matrix studied in Example 3 of Section 4.9.]
   b. Let \(x_0\) be any vector in \(\mathbb{R}^3\) with nonnegative entries whose sum is 1. (In Section 4.9, \(x_0\) was called a probability vector.) Explain why there are constants \(c_1, c_2, c_3\) such that \(x_0 = c_1 v_1 + c_2 v_2 + c_3 v_3\). Compute \(w^T x_0\), and deduce that \(c_1 = 1\).
   c. For \(k = 1, 2, \ldots\), define \(x_k = A^k x_0\), with \(x_0\) as in part (b). Show that \(x_k \to v_1\) as \(k\) increases.

28. [M] Construct a random integer-valued \(4 \times 4\) matrix \(A\), and verify that \(A\) and \(A^T\) have the same characteristic polynomial (the same eigenvalues with the same multiplicities). Do \(A\) and \(A^2\) have the same eigenvectors? Make the same analysis of a \(5 \times 5\) matrix. Report the matrices and your conclusions.
Diagonalize the matrices in Exercises 1–20, if possible. The eigenvalues are

\[
\begin{bmatrix}
\lambda_1 & 0 & 0 \\
0 & \lambda_2 & 0 \\
0 & 0 & \lambda_3
\end{bmatrix}
\]

For Exercises 1–8, use the matrix \( \lambda = \lambda_1 \) and one eigenvector is

\[
\begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix}
\]

and one eigenvector is

\[
\begin{bmatrix}
0 \\
1 \\
0
\end{bmatrix}
\]

Use the diagonalization theorem to find the eigenvalues of \( A \).

In Exercises 9–12, use the matrix \( \lambda = \lambda_2 \) and one eigenvector is

\[
\begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix}
\]

and one eigenvector is

\[
\begin{bmatrix}
0 \\
1 \\
0
\end{bmatrix}
\]

\[
\begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix}
\]

Use the diagonalization theorem to find the eigenvalues of \( A \).

In Exercises 13–16, use the matrix \( \lambda = \lambda_3 \) and one eigenvector is

\[
\begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix}
\]

and one eigenvector is

\[
\begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix}
\]

Use the diagonalization theorem to find the eigenvalues of \( A \).

Exercise 1. \( \lambda_1, \lambda_2, \lambda_3 \) are distinct, and \( A \) is diagonalizable.

Exercise 2. \( \lambda_1, \lambda_2, \lambda_3 \) are distinct, and \( A \) is diagonalizable.

Exercise 3. \( \lambda_1, \lambda_2, \lambda_3 \) are distinct, and \( A \) is diagonalizable.

Exercise 4. \( \lambda_1, \lambda_2, \lambda_3 \) are distinct, and \( A \) is diagonalizable.

Exercise 5. \( \lambda_1, \lambda_2, \lambda_3 \) are distinct, and \( A \) is diagonalizable.

Exercise 6. \( \lambda_1, \lambda_2, \lambda_3 \) are distinct, and \( A \) is diagonalizable.

Exercise 7. \( \lambda_1, \lambda_2, \lambda_3 \) are distinct, and \( A \) is diagonalizable.

Exercise 8. \( \lambda_1, \lambda_2, \lambda_3 \) are distinct, and \( A \) is diagonalizable.

Exercise 9. \( \lambda_1, \lambda_2, \lambda_3 \) are distinct, and \( A \) is diagonalizable.

Exercise 10. \( \lambda_1, \lambda_2, \lambda_3 \) are distinct, and \( A \) is diagonalizable.

Exercise 11. \( \lambda_1, \lambda_2, \lambda_3 \) are distinct, and \( A \) is diagonalizable.

Exercise 12. \( \lambda_1, \lambda_2, \lambda_3 \) are distinct, and \( A \) is diagonalizable.

Exercise 13. \( \lambda_1, \lambda_2, \lambda_3 \) are distinct, and \( A \) is diagonalizable.

Exercise 14. \( \lambda_1, \lambda_2, \lambda_3 \) are distinct, and \( A \) is diagonalizable.

Exercise 15. \( \lambda_1, \lambda_2, \lambda_3 \) are distinct, and \( A \) is diagonalizable.

Exercise 16. \( \lambda_1, \lambda_2, \lambda_3 \) are distinct, and \( A \) is diagonalizable.

Exercise 17. \( \lambda_1, \lambda_2, \lambda_3 \) are distinct, and \( A \) is diagonalizable.

Exercise 18. \( \lambda_1, \lambda_2, \lambda_3 \) are distinct, and \( A \) is diagonalizable.

Exercise 19. \( \lambda_1, \lambda_2, \lambda_3 \) are distinct, and \( A \) is diagonalizable.

Exercise 20. \( \lambda_1, \lambda_2, \lambda_3 \) are distinct, and \( A \) is diagonalizable.

Exercise 21. \( \lambda_1, \lambda_2, \lambda_3 \) are distinct, and \( A \) is diagonalizable.

Exercise 22. \( \lambda_1, \lambda_2, \lambda_3 \) are distinct, and \( A \) is diagonalizable.

Exercise 23. \( \lambda_1, \lambda_2, \lambda_3 \) are distinct, and \( A \) is diagonalizable.

Exercise 24. \( \lambda_1, \lambda_2, \lambda_3 \) are distinct, and \( A \) is diagonalizable.

Exercise 25. \( \lambda_1, \lambda_2, \lambda_3 \) are distinct, and \( A \) is diagonalizable.

Exercise 26. \( \lambda_1, \lambda_2, \lambda_3 \) are distinct, and \( A \) is diagonalizable.

Exercise 27. \( \lambda_1, \lambda_2, \lambda_3 \) are distinct, and \( A \) is diagonalizable.

Exercise 28. \( \lambda_1, \lambda_2, \lambda_3 \) are distinct, and \( A \) is diagonalizable.

Exercise 29. \( \lambda_1, \lambda_2, \lambda_3 \) are distinct, and \( A \) is diagonalizable.

Exercise 30. \( \lambda_1, \lambda_2, \lambda_3 \) are distinct, and \( A \) is diagonalizable.

Exercise 31. \( \lambda_1, \lambda_2, \lambda_3 \) are distinct, and \( A \) is diagonalizable.

Exercise 32. \( \lambda_1, \lambda_2, \lambda_3 \) are distinct, and \( A \) is diagonalizable.

Exercise 33. \( \lambda_1, \lambda_2, \lambda_3 \) are distinct, and \( A \) is diagonalizable.

Exercise 34. \( \lambda_1, \lambda_2, \lambda_3 \) are distinct, and \( A \) is diagonalizable.

Exercise 35. \( \lambda_1, \lambda_2, \lambda_3 \) are distinct, and \( A \) is diagonalizable.

Exercise 36. \( \lambda_1, \lambda_2, \lambda_3 \) are distinct, and \( A \) is diagonalizable.
### Eigenvalues and Eigenvectors

<table>
<thead>
<tr>
<th>Exercise</th>
<th>Matrix</th>
</tr>
</thead>
</table>
| 7. | \[
\begin{bmatrix}
1 & 0 \\
6 & -1
\end{bmatrix}
\] |
| 8. | \[
\begin{bmatrix}
5 & 1 \\
0 & 5
\end{bmatrix}
\] |
| 9. | \[
\begin{bmatrix}
3 & -1 \\
1 & 5
\end{bmatrix}
\] |
| 10. | \[
\begin{bmatrix}
2 & 3 \\
4 & 1
\end{bmatrix}
\] |
| 11. | \[
\begin{bmatrix}
-1 & 4 & -2 \\
-3 & 4 & 0 \\
-3 & 1 & 3
\end{bmatrix}
\] |
| 12. | \[
\begin{bmatrix}
4 & 2 & 2 \\
2 & 4 & 2 \\
2 & 2 & 4
\end{bmatrix}
\] |
| 13. | \[
\begin{bmatrix}
2 & 2 & -1 \\
1 & 3 & -1 \\
-1 & -2 & 2
\end{bmatrix}
\] |
| 14. | \[
\begin{bmatrix}
4 & 0 & -2 \\
2 & 5 & 4 \\
0 & 0 & 5
\end{bmatrix}
\] |
| 15. | \[
\begin{bmatrix}
7 & 4 & 16 \\
2 & 5 & 8 \\
-2 & -2 & -5
\end{bmatrix}
\] |
| 16. | \[
\begin{bmatrix}
0 & -4 & -6 \\
-1 & 0 & -3 \\
1 & 2 & 5
\end{bmatrix}
\] |
| 17. | \[
\begin{bmatrix}
4 & 0 & 0 \\
1 & 4 & 0 \\
0 & 0 & 5
\end{bmatrix}
\] |
| 18. | \[
\begin{bmatrix}
-7 & -16 & 4 \\
6 & 13 & -2 \\
12 & 16 & 1
\end{bmatrix}
\] |
| 19. | \[
\begin{bmatrix}
5 & -3 & 0 & 9 \\
0 & 3 & 1 & -2 \\
0 & 0 & 2 & 0 \\
0 & 0 & 0 & 2
\end{bmatrix}
\] |
| 20. | \[
\begin{bmatrix}
4 & 0 & 0 & 0 \\
0 & 4 & 0 & 0 \\
0 & 0 & 2 & 0 \\
1 & 0 & 0 & 2
\end{bmatrix}
\] |

In Exercises 21 and 22, \( A, B, P, \) and \( D \) are \( n \times n \) matrices. Mark each statement True or False. Justify each answer. (Study Theorems 5 and 6 and the examples in this section carefully before you try these exercises.)

21. a. \( A \) is diagonalizable if \( A = PD P^{-1} \) for some matrix \( D \) and some invertible matrix \( P \).
   
   b. If \( \mathbb{R}^n \) has a basis of eigenvectors of \( A \), then \( A \) is diagonalizable.
   
   c. \( A \) is diagonalizable if and only if \( A \) has \( n \) eigenvalues, counting multiplicities.
   
   d. If \( A \) is diagonalizable, then \( A \) is invertible.

22. a. \( A \) is diagonalizable if \( A \) has \( n \) eigenvectors.
   
   b. If \( A \) is diagonalizable, then \( A \) has \( n \) distinct eigenvalues.
   
   c. If \( AP = PD \), with \( D \) diagonal, then the nonzero columns of \( P \) must be eigenvectors of \( A \).
   
   d. If \( A \) is invertible, then \( A \) is diagonalizable.

23. \( A \) is a \( 5 \times 5 \) matrix with two eigenvalues. One eigenspace is three-dimensional, and the other eigenspace is two-dimensional. Is \( A \) diagonalizable? Why?

24. \( A \) is a \( 3 \times 3 \) matrix with two eigenvalues. Each eigenspace is one-dimensional. Is \( A \) diagonalizable? Why?

25. \( A \) is a \( 4 \times 4 \) matrix with three eigenvalues. One eigenspace is one-dimensional, and one of the other eigenspaces is two-dimensional. Is it possible that \( A \) is not diagonalizable? Justify your answer.

26. \( A \) is a \( 7 \times 7 \) matrix with three eigenvalues. One eigenspace is two-dimensional, and one of the other eigenspaces is three-dimensional. Is it possible that \( A \) is not diagonalizable? Justify your answer.

27. Show that if \( A \) is both diagonalizable and invertible, then so is \( A^{-1} \).

28. Show that if \( A \) has \( n \) linearly independent eigenvectors, then so does \( A^T \). [Hint: Use the Diagonalization Theorem.]

29. A factorization \( A = PDP^{-1} \) is not unique. Demonstrate this for the matrix \( A \) in Example 2. With \( D_1 = \begin{bmatrix} 3 & 0 \\ 0 & 5 \end{bmatrix} \), use the information in Example 2 to find a matrix \( P_1 \) such that \( A = P_1 D_1 P_1^{-1} \).

30. With \( A \) and \( D \) as in Example 2, find an invertible \( P_2 \) unequal to the \( P \) in Example 2, such that \( A = P_2 D P_2^{-1} \).

31. Construct a nonzero \( 2 \times 2 \) matrix that is invertible but not diagonalizable.

32. Construct a nondiagonal \( 2 \times 2 \) matrix that is diagonalizable but not invertible.

[M] Diagonalize the matrices in Exercises 33–36. Use your matrix program's eigenvalue command to find the eigenvalues, and then compute bases for the eigenspaces as in Section 5.1.

<table>
<thead>
<tr>
<th>Exercise</th>
<th>Matrix</th>
</tr>
</thead>
</table>
| 33. | \[
\begin{bmatrix}
-6 & 4 & 0 & 9 \\
-3 & 0 & 1 & 6 \\
-1 & -2 & 1 & 0 \\
-4 & 4 & 0 & 7
\end{bmatrix}
\] |
| 34. | \[
\begin{bmatrix}
0 & 13 & 8 & 4 \\
4 & 9 & 8 & 4 \\
8 & 6 & 12 & 8 \\
0 & 5 & 0 & -4
\end{bmatrix}
\] |
| 35. | \[
\begin{bmatrix}
11 & -6 & 4 & -10 \\
-3 & 5 & -2 & 4 \\
-8 & 12 & -3 & 12 \\
1 & 6 & -2 & 3
\end{bmatrix}
\] |
| 36. | \[
\begin{bmatrix}
8 & -18 & 8 & -14 \\
4 & 4 & 2 & 3 \\
0 & 1 & -2 & 2 \\
6 & 12 & 11 & 2
\end{bmatrix}
\] |