Test pile is being handed front-to-back. It is alphabetized by last name.

HW pile is being handed back-to-front.

Exam 1 average: 78.5%
So \( \text{vol} = \int_{a}^{b} (\int_{0}^{y} f(x,y) \, dx) \, dy \). To find \( A(x) = \int_{0}^{b} f(x,y) \, dy \), we find the cross-sectional area at \( x \). The total vol. of solid region is \( \int_{a}^{b} A(x) \, dx \). Call it \( A(x) \). The cross-sectional area is function of \( x \).
or, could find volume by first cross-sectioning parallel to the xz plane:

\[ V = \int_{c}^{d} \left( \int_{a}^{b} f(x,y) \, dx \right) \, dy \]

12.2: Double Integrals over non-rectangular regions

Rectangular region: \( \{(x,y) \mid a \leq x \leq b, c \leq y \leq d\} = [a,b] \times [c,d] \)
How would you describe this region?

\[ \{(x,y) \mid 1 \leq x \leq 2, \quad \frac{1}{2} x \leq y \leq \sqrt{x}\} \]

To integrate \( f(x,y) = x^2 + y^2 \) over this region:

\[ \int_{\frac{1}{2}x}^{x} \int_{\frac{1}{2}x}^{\sqrt{x}} (x^2 + y^2) \, dy \, dx \]

The point is that the limits of integration match the description of the region above.
Suppose we want to integrate 

$$f(x,y) = x^2 + y$$ 

over this region:

**Region D:** 

Bounded by: 

- $y = 1$, $y = 2$
- $y = x^2$, $y = 3 - x$

$$
\int \int_{D} (x^2 + y) \, dA = ?
$$

To describe D:

**Option 1:** 

$$\{(x,y) \mid 1 \leq x \leq 5, \quad \frac{g(x)}{4} \leq y \leq \frac{h(y)}{3} \}$$

**Option 2:** 

$$\{(x,y) \mid 1 \leq y \leq 2, \quad \sqrt{y} \leq x \leq \frac{y+3}{g(y)} \}$$

Option 2 is better; option 1 cannot be done with a single set description.

Remember: $dA$ is essentially shorthand for $dx \, dy$ or $dy \, dx$ (either order).
So \[ \iint_D x^2 + y \, dA \]

\[ = \int_0^1 \int_{y^3}^{y^2} (x^2 + y) \, dx \, dy \]

12.2 (b) Find volume of region under \( z = 2x + y^2 \), \& above reg. bounded by \( x = y^2 \), \( x = y^3 \).

\[ y^2 = y^3 \]
\[ 0 = y^2 - y^2 \]
\[ 0 = y^2(y-1) \]

\[ \{(x,y) \mid 0 \leq y \leq 1, y^3 \leq x \leq y^2 \} \]

\[ V_{\text{vol}} = \int_0^1 \left( \int_{y^3}^{y^2} (2x + y^2) \, dx \right) \, dy = \text{(next pg.)} \]
\[ \text{Fmer} = \int \left(2x + y^2\right) \, dx = \left[ x^2 + y^2 x \right]_{x=y^3}^{y^4} \]

\[ = \left[ y^4 + y^4 \right] - \left[ y^6 + y^5 \right] = 2y^4 - y^6 - y^5. \]

\[ \text{Avor} = \int_{0}^{1} \int_{0}^{2y^4} \frac{y^2 - 4y^3 - y^5}{y^6} \, dy \, dx = \left[ \frac{2y^5}{5} - \frac{4y^7}{7} - \frac{y^9}{9} \right]_{0}^{1} \]

\[ = \frac{2}{5} - \frac{4}{7} - \frac{1}{9} = \frac{84 - 30 - 35}{210} = \frac{19}{210} \]

\[ \int_{0}^{3} \int_{0}^{3y} e^{x^2} \, dx \, dy \]

Region of integration:

\[ \{(x, y) \mid 0 \leq y \leq 1, \ 3y \leq x \leq 3\} \]

\[ = \{(x, y) \mid 0 \leq x \leq 3, \ 0 \leq y \leq \frac{1}{3} x\} \]

redescribed with x on the "outside"
So \( (\text{intg.}) = \int_0^3 \left( \int_0^{\frac{-5x}{3}} e^{x^2} \, dy \right) \, dx \)

\[ \text{inner: } \int_0^{\frac{-5x}{3}} e^{x^2} \, dy = e^{x^2} y \bigg|_{y=0}^{y=\frac{-5x}{3}} = \frac{5}{3}xe^{x^2} \]

\[ \Rightarrow \int_0^3 \frac{5}{3}xe^{x^2} \, dx \]

\[ \text{outer: } \int_0^3 \frac{5}{3}xe^{x^2} \, dx \]

\[ = \frac{1}{3} \cdot \frac{5}{2}e^{x^2} \bigg|_0^3 = \frac{1}{6}(e^9 - e^0) = \frac{e^9 - 1}{6} \]