Exam 1 is next Thurs, in class. Some practice questions are posted. Those questions do cover every topic you'll be tested on.

more 11.6:

Consider a function \( f(x, y, z) \), and specific input \((a, b, c)\). Then \( f(a, b, c) = k \) (some value), and the set of all \((x, y, z)\) such that \( f(x, y, z) = k \) is a level set of \( f \), and \((a, b, c)\) is on that surface:
$f(x_1, y_1, z_1) = k+1$

level surface $f(x, y, z) = k$

$f(x_1, y_1, z_1) = k-1$

$\nabla f(a, b, c)$ is $\langle f_x(a, b, c), f_y(a, b, c), f_z(a, b, c) \rangle$

3-D vector.

$\nabla f(a, b, c)$ must be normal to the level surface $f(x_1, y_1, z_1) = f(a, b, c)$ at the point $(a, b, c)$.

(since $\nabla f$ tells you which way to move input to most rapidly increase the output of $f$.)
11.6 \(33\) \(z + 1 = xe^y \cos(z) \quad (4, 0, 0)\)

a) We don't have \(z = f(x, y)\) here.

So can't use

\[ z = f(a, b) + f_x(a, b)(x-a) + f_y(a, b)(y-b) \]

The given relation is equiv.

to \(1 = xe^y \cos(z) - z \).

\[ f(x, y, z) \]

Knew \((1, 0, 0)\) is on this level surface, and knew tangent plane at \((4, 0, 0)\) is:

\[-(x-1) + -(y-0) + -(z-0) = 0\]

For these, need normal vector.
use \( xe^y \cos z - z = 1 \)

\[ \nabla f : \nabla f = \nabla (xe^y \cos z - z) \]

\[ = \langle e^y \cos z, xe^y \cos z, -xe^y \sin z - 1 \rangle \]

\[ \nabla f(1,0,0) = \langle 1, 1, -1 \rangle \]

tan. plane at \((1,0,0)\) is

\[ 1(x-1) + 1(y-0) + -1(z-0) = 0 \]

\[ x+y-z = 1 \]
\((34)\)  \(\mathbf{v} = 2\mathbf{c}(x+z), (0,0,1)\)

a) \(\mathbf{v} = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z}\)

\[\nabla f = \left< \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z} \right>\]

\[= \left< \frac{1}{x+z}, -1, \frac{1}{x+z} - y \right>\]

\(\nabla f(0,0,1) = \left< 1, -1, 1 \right>\)

So the plane to given surface at \((0,0,1)\) is

\[1(x-0) + (-1)(y-0) + 1(z-1) = 0\]

pt. on plane

\[a(x-x_0) + b(y-y_0) + c(z-z_0) = 0\]

normal vector to plane
b)

"normal"

line's parametric eqns are:

\[ x = 0 + 1t \]
\[ y = 0 + (-1)t \]
\[ z = 1 + 1t \]
11.7 Local Extrema for 

\[ z = f(x, y) \]

- Local max
- Local min
- Neither local max nor local min, called saddle pt.

At any such points, \( f_x = 0 \) (assuming differentiability), \( f_y = 0 \)
\[ f_{xy} = \frac{1}{2y} \left( \frac{\partial^2 f}{\partial x^2} \right) \] represents "twisting."

\[ f_{xx} \] represents concavity in x-direction.
\[ f_{yy} \] represents concavity in y-direction.

Once critical points are found (where \( f_x = 0 \) & \( f_y = 0 \)), they can be tested to see whether they are local extrema using the 2nd derivative test:

Define "testing function"

\[ D(x,y) = f_{xx}(x,y) \cdot f_{yy}(x,y) - (f_{xy}(x,y))^2. \]

For a critical pt. \((a,b)\):
If \( D(a, b) \) is positive, then there is a local extremum there.

- If \( f_{xx} \neq f_{yy} \neq 0 \), \( \nabla \max \).
- If \( f_{xx} \neq f_{yy} = 0 \), \( \nabla \max \).

If \( D(a, b) \) is negative, then saddle point.

If \( D(a, b) = 0 \), no conclusion possible.
11.7 (4) \( f(x, y) = x^3y + 12x^3 - 8y \).

\[
\begin{align*}
\frac{\partial f}{\partial x} &= 3x^2y + 36x^2 = 0 \\
\frac{\partial f}{\partial y} &= x^3 - 8 = 0 \implies x = 2
\end{align*}
\]

\[3x^2(y + 12) = 0 \implies y = -12.\]

Only crit. pt. is \((2, -12)\).

\[
D = \left(\frac{\partial^2 f}{\partial x^2}\right) \left(\frac{\partial^2 f}{\partial y^2}\right) - \left(\frac{\partial^2 f}{\partial x \partial y}\right)^2
\]

\[D(2, -12) = (6xy + 72x)(0) - (3x^2)^2\]

\[= \text{negative.}\]

So saddle point at \((2, -12, -16)\).