SHOW APPROPRIATE WORK or EXPLANATION on each problem for full credit. Box or circle your final answers. **Answers need not be simplified.** Calculators/note sheets are NOT allowed. Numbers in the [ ] indicate what each problem is worth. Check the board for useful info.

1. For the curve \( \vec{c}(t) = \langle \sin(\frac{3}{2}t), t^{3/2}, \cos(\frac{5}{2}t) \rangle \), do the following:

   a) [6] Assuming this curve represents a particle's position, find the particle's velocity and acceleration at \( t = 9 \).

   \[
   \text{velociy} = \vec{c}'(t) = \langle \frac{3}{2} \cos(\frac{3}{2}t), \frac{3}{2} t^{1/2}, -\frac{5}{2} \sin(\frac{5}{2}t) \rangle \\
   \text{accel.} = \vec{c}''(t) = \langle -\frac{9}{4} \sin(\frac{3}{2}t), \frac{3}{4} t^{1/2}, -\frac{25}{4} \cos(\frac{5}{2}t) \rangle \\
   \]

   At \( t=9 \),
   \[
   \text{velociy} = \langle \frac{3}{2} \cos(\frac{3}{2} \cdot 9), \frac{3}{2} \cdot 9^{1/2}, -\frac{5}{2} \sin(\frac{5}{2} \cdot 9) \rangle = \langle 0, \frac{9}{2}, -\frac{5}{2} \rangle \\
   \text{accel.} = \langle -\frac{9}{4} \sin(\frac{3}{2} \cdot 9), \frac{3}{4} (9)^{1/2}, -\frac{25}{4} \cos(\frac{5}{2} \cdot 9) \rangle = \langle \frac{27}{4}, \frac{3}{4}, 0 \rangle 
   \]

   b) [8] Find the parametric equations of the line which is tangent to the given curve at the point \( (0, 8, 1) \).

   For \( \vec{c}(4) = \langle 0, 8, 1 \rangle \), need \( t^3/2 = 8 \Rightarrow t = 4 \). So:

   \[
   \vec{c}'(4) = \langle \frac{3}{2} \cos(\frac{3}{2} \cdot 4), \frac{3}{2} \cdot (4)^{1/2}, -\frac{5}{2} \sin(\frac{5}{2} \cdot 4) \rangle = \langle \frac{3}{2}, 3, 0 \rangle.
   \]

   So tan. line at \( (0, 8, 1) \) is:

   \[
   \left\{ \begin{array}{l}
   x = 0 + \frac{3}{2}t \\
   y = 8 + 3t \\
   z = 1 + 0t
   \end{array} \right. 
   \]

   c) [6] Sketch (as accurately as you can) the piece of curve from \( t = 0 \) to \( t = 4 \). Label initial and final points with their \((x, y, z)\) coordinates.

   \[
   \vec{c}(0) = \langle 0, 0, 1 \rangle \quad \vec{c}(4) = \langle 0, 8, 1 \rangle \\
   \vec{c}(1) = \langle 1, 1, 0 \rangle \quad \vec{c}(2) = \langle 0, 2 \sqrt{2}, -1 \rangle \\
   \vec{c}(3) = \langle 1, 3 \sqrt{3}, 0 \rangle 
   \]

   \[
   \text{y grows like this:} \\
   \text{looking from back,} \\
   \text{spiral towards as ccw.}
   \]

   So:

   \[
   \text{(Image of the spiral graph)}
   \]
2. For the function \( f(x, y) = \sqrt{y^2 + 2x} \), do the following:

a) [6] Find and graph the domain of \( f \) in the \( xy \) plane.

\[
y^2 + 2x \geq 0 \implies x = -\frac{1}{2} y^2
\]

b) [12] Draw a contour map of \( f \), showing level curves for \( z \)-levels of 0, 1, 2, 3. Label each contour with its \( z \)-level.

\[
\begin{align*}
\sqrt{y^2 + 2x} &= 0 \implies x = -\frac{1}{2} y^2 \\
\sqrt{y^2 + 2x} &= 1 \implies x = \frac{1}{2} - \frac{1}{2} y^2 \\
\sqrt{y^2 + 2x} &= 2 \implies x = 2 - \frac{1}{2} y^2 \\
\sqrt{y^2 + 2x} &= 3 \implies x = \frac{9}{2} - \frac{1}{2} y^2
\end{align*}
\]

3. [12] Given that \( w = \frac{xy}{z} \), \( x = e^{3t-r^2} \), \( y = rt^2 \), \( z = r \ln(t) \), use the chain rule to find an expression for \( \frac{\partial w}{\partial t} \).

You may leave a mixture of \( x \)'s, \( y \)'s, \( z \)'s, \( r \)'s and \( t \)'s in your answer.

\[
\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x} \frac{\partial x}{\partial t} + \frac{\partial w}{\partial y} \frac{\partial y}{\partial t} + \frac{\partial w}{\partial z} \frac{\partial z}{\partial t}
\]

\[
= \left( \frac{y}{z} \right) \left( 3e^{3t-r^2} \right) + \left( \frac{x}{z} \right) (2rt) + \left( -\frac{xu}{z^2} \right) \left( \frac{r}{t} \right)
\]
4. [12] Let \( f(x, y, z) = (z^3 - x^2)/y \) and let \( \vec{v} = (5, -2, 1) \). Find the directional derivative of the function \( f \) at the point \( (1, 2, 3) \) in the direction of \( \vec{v} \). That is, compute \( D_{\vec{v}}f(1, 2, 3) \), where \( \vec{u} \) is a unit vector in the same direction as \( \vec{v} \).

\[
\nabla f = \left\langle \frac{-2x}{y}, \frac{x^2 - z^3}{y^2}, \frac{3z^2}{y} \right\rangle, \quad \nabla f(1, 2, 3) = \left\langle -1, -\frac{12}{3}, \frac{27}{2} \right\rangle, \quad \vec{u} = \left\langle 5, 2, 1 \right\rangle.
\]

So \( D_{\vec{u}}f(1, 2, 3) = \nabla f(1, 2, 3) \cdot \vec{u} = \left\langle -1, -\frac{12}{2}, \frac{27}{2} \right\rangle \cdot \left\langle 5, -2, 1 \right\rangle = \frac{-5 + 13 + \frac{27}{2}}{\sqrt{30}} = \frac{43}{2\sqrt{30}}.

5. [12] Find an equation for the plane which is tangent to the surface \( x^2e^{x-3z} - 3yz = 8 \) at the point \( (3, \frac{1}{3}, 1) \).

\[
\nabla f = \left\langle (x^2 + 2x)e^{x-3z}, -3y, -3x^2e^{x-3z} - 3y \right\rangle
\]

\[
\nabla f (3, \frac{1}{3}, 1) = \left\langle 15e^0, -3, -27e^0 - 1 \right\rangle = \left\langle 15, -3, -28 \right\rangle.
\]

So tan. plane at \( (3, \frac{1}{3}, 1) \) is

\[
15(x-3) - 3(y-\frac{1}{3}) - 28(z-1) = 0.
\]
6. [12] Find all the critical points \((x, y)\) for the function \(f(x, y) = 3x^2y - 2y^3 - 3x^2\). (you should get 3 of them)

Hint: you should be able to factor the \(f_x(x, y) = 0\) equation, which should help

\[
\begin{align*}
    f_x &= 6yx - 6x = 0 \\
    f_y &= 3x^2 - 6y^2 = 0
\end{align*}
\]

\[
\begin{cases}
    6x(y-1) = 0 &\quad \Rightarrow \quad x = 0 \text{ or } y = 1 \\
    x^2 - 2y^2 = 0
\end{cases}
\]

\(x = \pm \sqrt{2} \quad y = 0\)

so critical pts. are \((0,0), (\sqrt{2}, 1), (-\sqrt{2}, 1)\)

7. [12] The function \(f(x, y) = 3x^2y + y^3 - 3x^2 - 3y^2 + 2\) has critical points at \((x, y) = (0, 0), (0, 2), (1, 1)\) and \((-1, 1)\).

Classify each point as a local maximum, local minimum, or saddle point, and fill in your answers in the table below.

Note that I’m also asking for the \(z\)-coordinate at each point.

<table>
<thead>
<tr>
<th>((x, y, z))</th>
<th>local max, min, or saddle?</th>
</tr>
</thead>
<tbody>
<tr>
<td>((0, 0, 2))</td>
<td>\text{max}</td>
</tr>
<tr>
<td>((0, 2, -2))</td>
<td>\text{min}</td>
</tr>
<tr>
<td>((1, 1, 0))</td>
<td>\text{saddle}</td>
</tr>
<tr>
<td>((-1, 1, 0))</td>
<td>\text{saddle}</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
    f_x &= 6yx - 6x \\
    f_{xx} &= 6y - 6 \\
    f_y &= 3x^2 + 3y^2 - 6y \\
    f_{yy} &= 6y - 6 \\
    f_{xy} &= f_{yx} = 6x \\
    D &= (6y-6)(6y-6) - (6x)^2 \\
    D(0, 0) &= (-6)(-6) - (0)^2 = 36 \quad \Theta = \text{max} \\
    D(0, 2) &= (6)(6) - (0)^2 = 36 \quad \Theta = \text{min} \\
    D(1, 1) &= (0)(0) - (6)^2 = -36 \quad \Theta \text{ saddle} \\
    D(-1, 1) &= (0)(0) - (-6)^2 = -36 \quad \Theta \text{ saddle}
\end{align*}
\]