Final Exam is Wed, 12/16, 7-9 pm, in FSHN 101. It's comprehensive.

Bring I.D. You will be provided with a sheet of formulas. Review materials are posted. Solutions soon.
14.7 Stokes' Thm

\[ \int_C \mathbf{F} \cdot d\mathbf{r} = \oint_S \text{curl}(\mathbf{F}) \cdot d\mathbf{s} \]
14.8 Divergence Thm:

\[ \int_{\partial E} \mathbf{F} \cdot d\mathbf{s} = \int_{\Omega} \nabla \cdot \mathbf{F} \, d\Omega = \iint_{\Sigma} \mathbf{F} \cdot \mathbf{n} \, dS = \iiint_{E} \operatorname{div} \mathbf{F} \, dV \]
14.8 \hspace{1cm} (18) \hspace{1cm} \vec{F} = \langle x^2, 2x^2, y^2 \rangle

\( S' \) is surface of cube \([0,1] \times [0,1] \times [0,1]\):

\( E \)

\( \text{(net outward flux of } \vec{F} \text{ across } S') \)

\[ = \iint_{S'} \vec{F} \cdot \hat{n} \, dS \]

\( S' \) outward unit normal
\[
\iiint_E \text{div}(\mathbf{F}) \, dV = \int_0^1 \int_0^y \int_0^2 x \, dz \, dy \, dx
\]

\[
\text{div}(\mathbf{F}) = \nabla \cdot \mathbf{F} = \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right) \cdot (x^2, 2zx, y^2)
\]

\[
= \frac{\partial}{\partial x}(x^2) + \frac{\partial}{\partial y}(2zx) + \frac{\partial}{\partial z}(y^2)
\]

\[
= 1
\]

\[
\mathbf{F} = \langle x, y, z \rangle, \quad S:
\]

\[
\iint_S \mathbf{F} \cdot d\mathbf{S} = \iiint_E \text{div}(\mathbf{F}) \, dV
\]

\[
S: z = 4 - x^2 - y^2, \quad \text{bottom} \quad z = 0
\]

\[
\text{top} \quad z = 4 - x^2 - y^2
\]

\[
\text{cross section} \quad \frac{\pi}{4}
\]
\[ = \int_0^\pi \int_0^2 \int_0^{y-r^2} 3 \quad r \, dz \, dr \, d\theta \]

\[ = \cdots = 0. \]

14.7 (13) \[ \vec{F} = \langle x^2z^2, y, 2z^2 \rangle \]

\[ z = y - x - y = f(x,y) \]

**base region** \( R \):
$$\oint \mathbf{F} \cdot d\mathbf{r} = \iint \text{curl}(\mathbf{F}) \cdot d\mathbf{s}$$

$$f(x,y) = x^2 - y$$

$$\mathbf{r} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \mathbf{F}$$

$$\left| \begin{array}{ccc} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 & y & z^2 \end{array} \right| = \langle -4z, -4x, 0 \rangle$$

$$= \int_0^4 \int_0^{4-x} \langle 0, -4z, 0 \rangle \cdot \langle 1, 1, 1 \rangle \, dy \, dx$$

$$= \int_0^4 \int_0^{4-x} -4z \, dy \, dx = \int_0^4 \int_0^{4-x} -4(4-x-y) \, dy \, dx$$

$$= \ldots = 0.$$
19.7 (6) \[ \vec{F} = \langle 0, -x, y \rangle \]

\[ \oint_c \vec{F} \cdot d\vec{r} \quad ? \quad \iint_S \text{curl} \vec{F} \cdot d\vec{s} \]

LHS

RHS
LHS: \[ C : \overrightarrow{r}(t) = \langle 2\cos(t), 2\sin(t), 0 \rangle, \quad 0 \leq t \leq 2\pi \]

\[ \oint_{C} \overrightarrow{F} \cdot d\overrightarrow{r} = \int_{0}^{2\pi} \langle 0, -2\cos(t), 2\sin(t) \rangle \cdot \langle -2\sin(t), 2\cos(t), 0 \rangle \, dt \]

\[ = \int_{0}^{2\pi} \left( -4 \cos^2(t) \right) \, dt = -2 \int_{0}^{2\pi} \left( 1 + \cos(2t) \right) \, dt \]

\[ = -2 \left( 2\pi \right) = -4\pi. \]

RHS: \[ \iint_{\Sigma} \text{curl} \overrightarrow{F} \cdot d\overrightarrow{S} = \ldots \quad \text{should} = \]

I'll finish this now. \( J \) class ended here.
\[
= \int \int \mathbf{1}_{0, -17} \cdot \left( \frac{x}{\sqrt{14-x^2-y^2}}, \frac{y}{\sqrt{14-x^2-y^2}}, 1 \right) \, dA
\]

\[S: \ z = \sqrt{14-x^2-y^2}, \quad f_x = \frac{-x}{\sqrt{14-x^2-y^2}}, \quad f_y = \frac{-y}{\sqrt{14-x^2-y^2}}\]

\[\mathbf{r} = \mathbf{f}_x \times \mathbf{f}_y, \quad dS \sim \mathbf{f}_x \times \mathbf{f}_y \, dA\]

Polar cords: \[\theta \in [0, \pi], \quad r \in [0, 2]\]

\[
= \int_0^{2\pi} \int_0^2 \left( \frac{r \cos \theta}{\sqrt{14-r^2}} - 1 \right) r \, dr \, d\theta.
\]

I'll change order because it's going to help:
\[(\text{above}) = \int_0^r \int_0^{2\pi} \left( \frac{r^2 \cos \theta}{r^4 - r^2} - r \right) \, d\theta \, dr\]

\[\text{Inner} = \left[ \frac{r^2}{14-r^2} \sin \theta - r \theta \right]_0^{2\pi} = -2\pi r.\]

\[\text{Outer} = -\pi r^2 \bigg|_0^2 = -4\pi.\]