1. Find parametric equations for the line which is tangent to the curve \( \vec{r}(t) = (3t, t \ln(t), \sqrt{t}) \) at the point where \( t = 1 \).

2. Find an equation for the plane which is tangent to the surface \( x \sin(4y - 3z) - \frac{y^2}{x} = 6 \) at the point \((-1, \frac{3}{2}, 2)\).

3. Compute the directional derivative of \( f(x, y, z) = xe^{-x^2 - y^2} \) at the point \((1, 0, 2)\) in the direction of \( \vec{u} = (1, -1, 3) \). That is, compute \( D_{\vec{u}}f(1, 0, 2) \), where \( \vec{u} \) is a unit vector in the same direction as \( \vec{u} \). What would you change \( \vec{u} \) to in order to get the largest possible value of \( D_{\vec{u}}f(1, 0, 2) \)? What would this maximum value of \( D_{\vec{u}}f(1, 0, 2) \) be?

4. Given that \( w = xy^2z^3 + \sqrt{y} \), \( x = t^2 \), \( y = \ln(t) + t \), and \( z = \frac{1}{t^2} \), use the chain rule to find \( \frac{dw}{dt} \).

5. The function \( f(x, y) = y(x - 2)^3 + y^2 - y \), has exactly two critical points. One is a saddle point at \((x, y) = (2, \frac{1}{4})\). Find the other critical point, and classify it (as local max, local min, or saddle).

6. Compute \( \int \vec{F} \cdot d\vec{r} \), where: a) \( \vec{F}(x, y) = (2y, 1 - x) \) and \( C \) is the piece of the curve \( y = x^2 \) from \((-1, 1)\) to \((2, 4)\); b) \( \vec{F}(x, y) = (x - y, 2y - x) \) and \( C \) is given by \( \vec{r}(t) = (t, 5 - t^2), \ 0 \leq t \leq 2 \).

7. Compute the volume of the solid which lies above the disc \( x^2 + y^2 \leq 1 \) and below the surface \( z = 1 - (x^2 + y^2)^{1/4} \).

8. Let \( S \) be the piece of the surface \( z = 4 - x^2 - y \) which lies in the first octant, and let \( E \) be the solid region bounded by \( S \) and the coordinate planes. Sketch \( S \), and then set up iterated integrals (limits and all) for calculating the area of \( S \) and the volume of \( E \).

9. a) Evaluate the triple integral \( \int_0^1 \int_0^1 \int_0^{2-x^2} 1 \, dz \, dy \, dx \). b) Give a complete description and/or sketch of the region whose volume is represented by the integral in part(a).

10. Evaluate the triple integral \( \iiint_E z \, dV \), where \( E \) is the region above \( z = \sqrt{x^2 + y^2} \) and below \( x^2 + y^2 + z^2 = 9 \).

11. Let the region \( R \) be the region in the \( xy \) plane bounded by \( y = x, y = 3x, xy = 1 \) and \( xy = 2 \). Sketch \( R \), and then use the transformation \( u = \frac{y}{x}, \ v = xy \) to evaluate \( \iint_R x \, dA \).

12. In each case, determine whether the given field is conservative. If it is, find a potential function \( f \) such that \( \vec{F} = \nabla f \).
   a) \( \vec{F}(x, y, z) = (2xz, 2x - y, x^2 + z) \)
   b) \( \vec{F}(x, y, z) = (2xz, -y, x^2 + z) \)

13. For each given field and curve, use Greene's Theorem to help you compute \( \oint_C \vec{F} \cdot d\vec{r} \).
   a) \( \vec{F}(x, y) = (-xy, x^2) \), \( C \) is as shown:
   
   b) \( \vec{F}(x, y) = (\sin(x) - y^2, x + e^{-y}) \), \( C \) is as shown:

14. Refering to the illustration below, do the following two problems:

   Use the Divergence Theorem to help you compute \( \iiint_E \vec{F} \cdot d\vec{S} \), where \( \vec{F}(x, y, z) = (x^2, y, 2z) \) and \( S \) is the surface of the solid \( E \) sketched below.

   Use Stokes' Theorem to help you compute \( \oint_C \vec{F} \cdot d\vec{r} \), where \( \vec{F}(x, y, z) = (yz, xz, 2xy) \) and \( C \) is the curve shown below. Note that \( C \) lies in the surface \( z = \sqrt{y} \).

   \( E \) is the solid region lying below the surface \( z = \sqrt{y} \) and above the rectangle \([0, 2] \times [0, 4] \).

   \( S \) is the entire surface of \( E \).

   \( C \) is the closed curve which bounds the top surface.
**Fundamental Theorem for Line Integrals:**

Let $C$ be a piecewise-smooth, continuous curve with initial point $\vec{x}_0$ and final point $\vec{x}_1$. If $\nabla f$ is continuous on $C$, then

$$\int_C \nabla f \cdot d\vec{r} = f(\vec{x}_1) - f(\vec{x}_0).$$

**Greene's Theorem:**

Let $C$ be a piecewise-smooth, positively-oriented, simple curve in the plane and let $D$ be the region bounded by $C$. If $\vec{F} = (P, Q)$ is continuously differentiable on an open region containing $D$, then

$$\int_C \vec{F} \cdot d\vec{r} = \iint_D \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA.$$

**Stokes' Theorem:**

Let $S$ be a piecewise-smooth, oriented surface in space and let the boundary of $S$ be a piecewise-smooth simple closed curve $C$, with orientation consistent (by the right-hand rule) with that of $S$. If $\vec{F} = (P, Q, R)$ is continuously differentiable on an open region which contains $S$, then

$$\int_C \vec{F} \cdot d\vec{r} = \iint_S \text{curl} \vec{F} \cdot d\vec{S}.$$

**Divergence Theorem:**

Let $S$ be a piecewise-smooth, closed surface in space, oriented outward. Let $E$ be the solid region enclosed by $S$. If $\vec{F} = (P, Q, R)$ is continuously differentiable on an open region which contains $E$, then

$$\iint_S \vec{F} \cdot d\vec{S} = \iiint_E \text{div} \vec{F} \, dV.$$

For a parameterized curve $\vec{r}(t) = (x(t), y(t), z(t))$, $a \leq t \leq b$:

- $\vec{r}'(t) = \text{tangent vector}$
- $\vec{T} = \frac{\vec{r}'(t)}{|\vec{r}'(t)|} = \text{unit tangent vector}$
- $s(t) = \int_a^t |\vec{r}'(t)| \, dt = \text{arc length function}$

$$ds = |\vec{r}'(t)| \, dt \quad \quad d\vec{r} = \vec{T} \, ds = (dx, dy, dz) = \vec{r}'(t) \, dt = (x'(t), y'(t), z'(t)) \, dt.$$

For a surface $S$:

$$d\vec{S} = \vec{n} \, dS,$$

where $\vec{n}(x, y, z)$ is a unit normal vector to $S$ at $(x, y, z)$, and $dS$ is a differential area element of $S$ at $(x, y, z)$.

If $S = \{(x, y, z) \mid z = f(x, y), \ (x, y) \in D\}$, then $dS = \sqrt{f_x^2 + f_y^2 + 1} \, dA$. 

**Polar/cylindrical coordinates:**

- $x = r \cos(\theta)$
- $y = r \sin(\theta)$
- $z = z$
- $r^2 = x^2 + y^2$
- $\tan(\theta) = y/x$
- $dA = r \, dr \, d\theta$
- $dV = r \, dr \, d\theta \, dz$

**Spherical coordinates:**

- $x = \rho \sin(\phi) \cos(\theta)$
- $y = \rho \sin(\phi) \sin(\theta)$
- $z = \rho \cos(\phi)$
- $\rho^2 = x^2 + y^2 + z^2$
- $\rho^2 + z^2 = r^2 + z^2$
- $dV = \rho^2 \sin(\phi) \, d\rho \, d\theta \, d\phi$

**Jacobian:**

$$\frac{\partial(x, y)}{\partial(u, v)} = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}$$

**div $\vec{F}$:**

$$\text{div} \vec{F} = \nabla \cdot \vec{F}$$

**curl $\vec{F}$:**

$$\text{curl} \vec{F} = \nabla \times \vec{F}$$
1. \( \mathbf{r}'(t) = \langle 3, 1 \rangle + \langle 1, 1 \rangle t \), so \( \mathbf{r}'(1) = \langle 3, 1 \rangle + 1 \). Also \( \mathbf{r}'(1) = \langle 3, 1 \rangle \), so line is

\[
\begin{align*}
  x &= 3 + 3t \\
  y &= 0 + t \\
  z &= 1 + \frac{t}{2}
\end{align*}
\]

2. \( x \sin(4y-3z) - \frac{y^2}{x} = 6 \). This is a level surface of \( f \), so normal vector is given by

\[
\nabla f = \langle \sin(4y-3z) + \frac{4y^2}{x^2}, 4x \cos(4y-3z) - \frac{2}{x}, -3x \cos(4y-3z) - \frac{2y^2}{x} \rangle,
\]

and \( \nabla f (1,3,2) = \langle 6, 6, 0, 9 \rangle \), so plane is \( 6(x-1) + 6(y-3) + 9(z-2) = 0 \), or \( 2x + 3y + 3z = 14 \).

3. \( \nabla f = \langle -2x e^{-x^2-y^2}, -2y e^{-x^2-y^2}, e^{-x^2-y^2} \rangle = \langle -2x, -2y, 1 \rangle \), \( \nabla f(1,0) = \langle -2, 0, 1 \rangle \).

4. To maximize \( \Delta_t f(1,0) \), choose \( \mathbf{u} = \frac{\nabla f(1,0)}{\| \nabla f(1,0) \|} = \frac{\langle -2, 0, 1 \rangle}{\sqrt{5}} \). Max \( \Delta_t f(1,0) = |\nabla f(1,0)| = \sqrt{5} \).

5. \( f_x = 3y(x-2)^2 \) \( f_y = (x-2)^2 + y-1 \) \( f_z = \begin{cases} 0 & y = 0 \text{ or } x = 2 \\ x-2 & x > 2 \end{cases} \), \( f = \begin{cases} 3y(x-2)^2 & y = 0 \text{ or } x < 2 \\ (x-2)^2 + y-1 & x > 2 \end{cases} \)

So critical pts. are \((2,1,3)\). Test for this is \( D(\text{two variables}) = f_{xx} f_{yy} - f_{xy}^2 = [6y(x-2)]^2 - [3x(x-2)]^2 \).

So: \( D(2,1) = 0 \), critical pt. \((2,1,3)\) is a saddle.

6. One way: \( \mathbf{r}'(t) = \langle 1, -1, 2 \rangle dt \), so get \( \int_0^2 \langle t - (t-5) + (t-2) \rangle dt = \int_0^2 (t^2 - 3t^2 - 7t + 5) dt = [\frac{t^3}{3} - \frac{3t^2}{2} - \frac{7t}{2}]_0^2 = -24 \).

Other way: \( x - y - z = 0 \), we have \( \int_0^2 f = \frac{1}{2} \left( \int_0^2 x^2 + y^2 \right) \) initial pt.

7. In cylindrical coords, \( x = r \cos \theta \), \( y = r \sin \theta \), \( z = z \), surface is \( z = 1 - \sqrt{r} \).

So volume = \( \int_0^{2\pi} \int_0^1 (1 - \sqrt{r}) r dr d\theta = 2\pi \int_0^1 \left( \frac{r^2}{2} - \frac{r^{3/2}}{3} \right) \).

8. \( z = 0 \Rightarrow y = 4x^2 \), \( x = 0 \Rightarrow z = 4y, \ y = 0 \Rightarrow z = 4x^2 => dS = \sqrt{\frac{x^2}{y^2} + \frac{y^2}{z^2} + 1} \).

Area = \( \int_0^1 \int_0^2 \sqrt{x^2 + y^2 + z^2} \) \( dA = 2\pi \int_0^1 \left( 2 - x^2 + x^2 \right) dx = 14 \).

9. a) Inner: \( \int_0^1 xy dz = \left( \frac{1}{2} \right) \left( 0^2 \right) = 0 \);

Middle: \( \int_0^1 (2x^2 - x^3) dy = \left( 2x^2 - x^3 \right) \left( -2x^2 + x^3 \right) = \left( 2 - 2x + x^2 \right) \)

Outer: \( \int_0^1 (2x^2 - x^3) dy = \left( \frac{11}{12} \right) \).

b) \( 0 \leq x \leq 1, \ 0 \leq y \leq 1 - x \):

\[
\int_0^1 \int_0^{1-x} dy \int_0^x dz = \int_0^1 \left( 1 - x \right) dx = \frac{1}{2} \cdot \frac{11}{12} = \frac{11}{24}.
\]

10. Using spherical coords:

\[
\int_{\rho=0}^{\rho=1} \int_{\phi=0}^{\phi=\pi} \int_{\theta=0}^{\theta=2\pi} \rho^2 \sin \theta d\theta d\rho d\phi = 2\pi \left( \frac{9}{4} \right) \int_0^\pi \sin \phi d\phi = \frac{81}{8} \pi.
\]
\[ \begin{aligned}
&\begin{aligned}
&\nabla \times \mathbf{F} = \begin{vmatrix}
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\
2x & z & 2x \\
x^2 & y & x^2 + z
\end{vmatrix} = (0, 2x \cdot 2x, 2) = \mathbf{0}, \text{ so not conservative.}
\end{aligned}

&\begin{aligned}
&\nabla \times \mathbf{F} = \begin{vmatrix}
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\
0 & 2x & 2z \\
x^2 & y & x^2 + z
\end{vmatrix} = (0, 2x \cdot 2x, 0), \text{ so conservative.}
\end{aligned}
\end{aligned} \]

\[ \begin{aligned}
&\begin{aligned}
&\nabla \cdot \mathbf{F} = \int_{C} \frac{1}{2\pi} \sqrt{1 + \left( \frac{x}{y} \right)^2} d\theta = \int_{0}^{\pi} \frac{1}{2\pi} \sqrt{1 + \left( \frac{x}{y} \right)^2} d\theta = \frac{1}{2\pi} \left[ \frac{2\pi}{3} \right] = \frac{1}{3}.
\end{aligned}
\end{aligned} \]

\[ \begin{aligned}
&\text{First case:} \quad \iint_{E} \mathbf{F} \cdot d\mathbf{S} = \iint_{E} \nabla \cdot \mathbf{F} \, dV = \iiint_{E} \left( \frac{x}{y} + 1 \right) \, dV = \iiint_{E} \left( x^2 + 1 \right) \, dV = \iiint_{E} \left( x^2 + 1 \right) \, dV = \frac{4}{3} \cdot 10 = \frac{40}{3}.
\end{aligned} \]

\[ \begin{aligned}
&\text{Second case:} \quad \iint_{E} \mathbf{F} \cdot d\mathbf{S} = \iiint_{E} \left( \nabla \cdot \mathbf{F} \right) \, dV = \iiint_{E} \left( \frac{1}{y} \right) \, dV = \iiint_{E} \left( \frac{1}{y} \right) \, dV = \frac{1}{3} \cdot \frac{16}{3} = \frac{16}{9}.
\end{aligned} \]