I want to note that the Chain Rule can be used to "generalize" basic rules:

\[ \sin x, \cos x, \tan x, \sec x \]

on Exam 3 know deriv for

Early exam by approval only. Email Remaley.

Sections 12, 15 take it in Todd 150.

Sections 3, 6 take it in Todd 116.

Exam 4 is next Tues, 9/27, 6-7:15 pm.
The chain rule.

If \( f \) is derivable and \( g \) is derivable, then the derivative of \( f \circ g \) is given by

\[
(f \circ g)'(x) = f'(g(x)) \cdot g'(x)
\]

From 2.6, 2.7: Both applications of

\[
\frac{dx}{dp} = n x^n - 1
\]

\[
\sin(x) = \frac{dx}{dp}
\]

\[
\cos(x) = \frac{dx}{dp}
\]

\[
\sinh(x) = \frac{dx}{dp}
\]

\[
\cosh(x) = \frac{dx}{dp}
\]

**Generalized**
Now take $f(x)$ (both sides):

\[
\sin (x + f(x)) = 2x + f(x) \]

since $f(x)$ and $g(x)$ become $f(x)$, you can then $g(x)$ as an unknown.

This curve, can have $y$ as an unknown.

To find on exercise for the search of $x$:

\[
\exists \quad \sin (x+y) = 2x + y
\]

Be solved for $\sin (x+y)$.

Note that $y$ cannot be solved in a way that $y$ can not.

Equation 2.6: Sometimes $x$ and $y$ are
\[
\begin{align*}
x(x), f &= \frac{1 - \cos(x + y)}{\cos(x + y) - 2} \\
(1 - \cos(x + y)) \left(1 - \frac{1}{\cos(x + y)}\right) &= 2 + f(x) \\
\cos(x + y) + \cos(x + y) &= 2 + f(x) \\
\cos(x + y) &= 2 + f(x)
\end{align*}
\]

Use algebra to solve for \( f(x) \):
\[
1 + f(x) = 2 + f(x)
\]
2.7 Idea: Sometimes variables are related to each other, but they are also changing with time.

Ex: \( x = \frac{4}{3} \pi r^3 \) vol. of sphere.

Ex: \( x = \frac{4}{3} \pi (t+7)^3 \)

Also the volume of a cube \( A \times t \) changes with time. Thus "rate relation" can take \( \frac{d}{dt} \) (both sides).
\[
\lim_{x \to a^+} f(x) = \text{def. if } f \text{ is cont. at } x=a
\]

\[
\frac{x-a}{x-a} = 1
\]

\[
\lim_{x \to a^+} f(x) = f(a)
\]

Define the Squeeze Theorem test.
\[ \lim_{x \to -2^+} f(x) = -\infty. \]

\[ \lim_{x \to 1^-} f(x) = 3. \]

Pos. neg. zero

\[ f(0) \text{ appears to be:} \]

\[ x \to \infty \]

Since \[ \lim_{x \to 1} f(x) = f(3) = 12 \]

Is \( f \) left or right continous at \( x = 3 \) ?

Where is \( f \) discontinuous? \( x = 3 \) \( x = 3. \)

\[ f \]

\[ 3 \quad 1 \quad -2 \]

\[ \text{Figure} \]

\[ \text{Diagram} \]
\[ f(x) = \begin{cases} x - 2x + 1 & \text{if } x < 0 \\ 2x - x^2 + 1 & \text{if } x \geq 0 \end{cases} \]
\[
\lim_{x \to 3} \frac{(x+3)(x+2)}{x^2-x-6} = \frac{x+2}{x-3} = \frac{5}{5} = 1.
\]

Don't write stuff like:

\[
\lim_{x \to 3} (x+2) = 5.
\]

Red:

\[
\lim_{x \to 3} (x+2) \rightarrow 5.
\]

As \( f \) is discontinuous at \( x = 3 \),

\[
\text{if (3) is undefined.}
\]
\[
\lim_{h \to 0} \frac{\sqrt{n+x+h} - \sqrt{n+x}}{h} = \lim_{h \to 0} \frac{\sqrt{n+x+h} - \sqrt{n+x}}{h} \\
= \frac{f(x+h) - f(x)}{h}
\]

If \( f(x) = \sqrt{x} \),

\[
1 = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}
\]

Def: \( f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \)