4.3 Sketch a graph that satisfies all:

\[ f(0) = f(12) = f(14) = 0 \]

Exams 2 will cover through 4.14.

Material Friday. I will post it today.

I forgot to post Exam 2 review.
\[ f(c) = 0. \]

of (or at least one number of all \( x \)).

Then there must be at (or in \([a, b]\)).

Suppose \( f \) is continuous on \([a, b]\).

Then, there exists a number \( c \) in \((a, b)\), and

**Rolle's Theorem**
as in Rolle's

\[ f'(c) = 0 \]

Note: \[ f'(a) = f'(b) \]

\[
\frac{b - a}{f(b) - f(a)} = f'(c)
\]

There is at least one \( c \) in \( (a,b) \).

Then there is at least one \( c \) in \( (a,b) \).

\( \text{cont. on [a,}\, b]\) and differentiable on \( (a,b) \).

\[ \text{Mean Value Theorem: Suppose } f \]

\[ \text{is } \]
Now, find all $c$ in $(-1,1)$ such that $f'(c) = \frac{f(1) - f(-1)}{1 - (-1)}$.

Is $f$ differentiable on $(-1,1)$? Yes, because it's a polynomial, or because $f$ is a polynomial function.

Is $f$ cont. on $(-1,1)$? Yes, because $f(x) = 6x^2 + x^3$.

$x = -\frac{\sqrt{2}}{2}$,
$17\) Show (prove) that the equation

$$1 + 2x + x^3 + 4x^5 = 0$$

has exactly one real root.

\[ \frac{c + 2 = 2}{6c + 2 = 6} \]

which (in this problem) is:

\[ \frac{f(1) - f(-1)}{10 - 6} = \frac{1}{2} \]
Thus by Rolle's Theorem, there must be a critical point in the interval $(-1, 0)$ where $f'(c) = 0$. Let's say $f(x_1) = 0$ and $f(x_2) = 0$.

Now, suppose the givenLisae were true e.

a critical in $(1, 0)$ where $f(c) = 0$.

Then, there must be a critical in $(-1, 0)$, so by $x = 1 - 2 - 1 - 4 = -6$

Therefore, $f(0) = 1$

Thus, $f(x) = 1 + 2x + x^2 + 9x^5$.
ver than one zero.

impossible. Therefore, f cannot have

\[ z + 36z^2 + 206z = 0 \]

Put this equals

\[ 5(c, b) = 0 \]

a number b between x, & x2 where

1/3
Thus \( f(x) = g(x) + C \) for all \( x \in (a, b) \).

Then \( f(x) \) is constant on \((a, b)\).

Thus \( f(x) = 0 \) for all \( x \in (a, b) \).

Finally, based on the MVT:
\[ \frac{f(x)}{f(x)} = 1 \quad \text{for all } x \text{ in domain of } f. \]

\[ f(x + T) = f(x) \]

The function is periodic with period \( T \).

A few more ideas:

1. Even graphs
2. Odd graphs
\[
\begin{align*}
|u(x)| &= \sin(e^x) \quad \text{is not periodic at all.} \\
|g(x)| &= e^{|\sin(x)|} \\
T &= \frac{\pi}{2} \\
f(x) &= \sin^2(x) \\
L &= T
\end{align*}
\]