\[ y = \frac{\sin^2(x) + \ln(y)}{\sqrt{x + 1}} \]

Ex. 3.3② Find y for

3) Find y using implicit differentiation.

2) Break down the ln (natural logarithm) part.

1) Take ln of both sides.

Sometimes, you can make finding a

more 3.3
\[ \frac{(x^2 + 1)^2}{\sin x + \frac{1}{x}} \cdot \left[ \frac{1}{1 + x^2} - \frac{\sec^2 x}{x^2} \right] = \sqrt{y} \]

\[ f \left( \frac{2 + x^2}{x^2} \right) \cdot f(1) = \sqrt{y} \]

\[ \frac{\tan x}{\sec x} + 4 \cdot \sin^2 x \]

\[ \frac{1}{2} \cdot \cos x \]

\[ \frac{2 + x^2}{x^2} \cdot \frac{1}{\tan x + \sec x} = \frac{2}{x^2} \]

\[ \frac{d}{dx} : \frac{g}{h} = 2 \ln (\tan x + \sec x) - 2 \ln (x^2 + 1) \]

\[ \text{limit :} \quad \lim_{x \to 0} \frac{2 + x^2}{x^2} \cdot \frac{1}{\tan x + \sec x} = \sqrt{y} \]

\[ \ln (\tan x + \sec x) - 2 \ln (x^2 + 1) \]

\[ \ln (13) : \ln y \]

\[ \ln (13) = \ln y \]
Another: \( \textcircled{50} \)

Three rules do not apply: \( \frac{d}{dx} \left( e^x \right) = e^x \) \( \text{cost} \) \( \frac{d}{dx} \sqrt[3]{x^3} = x \)

Use logarithmic diff:

\[
\frac{dy}{dx} = \frac{7x^2}{\cos(x)} \cdot \ln(\cos(x))
\]

\[
\frac{dy}{dx} = \cos(x) \cdot \frac{7x^2}{\ln(\cos(x))}
\]

\[
\frac{d}{dx} \left( \frac{1}{x^3 \sin(x)} + \frac{\tan(x)}{3} \left( x - 1 \right) \right)
\]
The trig functions, being periodic, are not actually invertible:

3.5: Inverse Trig Functions

\[ \int \frac{x}{\cos(x)} - \sin(x) \, dx = \]

\[ x - \frac{x}{\cos(x)} \cdot \sin(x) \]
\[ y = \sin(x), \]

If \( \sin(x) \)

is not one-to-one, there exists a region where \( \sin(x) \) is not restricted.

To get an inverse function, restrict the domain of \( \sin(x) \) to a region where \( \sin(x) \) is one-to-one. The inverse functions of restricted \( \sin(x) \) are actually inverse trigonometric functions.
\[ y = \sin^{-1}(x) \]

The domain is \([\frac{-\pi}{2}, \frac{\pi}{2}]\).

Invert the restricted sine function.
So: \( \sin^{-1} (\sin(x)) = x \) only for \( x \) in \([-\frac{\pi}{2}, \frac{\pi}{2}]\).

Recall: \( f(f^{-1}(x)) = x \) for all \( x \) in \( \text{dom}(f) \).

\[
f^{-1}(f(x)) = x \text{ for all } x \in \text{dom}(f^{-1})
\]

Examples:

\[
sin^{-1}(0) = \frac{\pi}{2}
\]

\[
sin^{-1}(-1) = -\frac{\pi}{2}
\]

\[
f^{-1}(\frac{\pi}{2}) = 0
\]
\[ \sin^{-1}(\sin(\frac{\pi}{3})) = \frac{\pi}{3}. \]
\[ y = \tan^{-1}(x) \]

\[ \cos^{-1}(\cos(x)) = x \text{ for all } x \text{ in } [-1, 1] \]

\[ \cos(\cos^{-1}(x)) = x \text{ for all } x \text{ in } [-1, 1] \]

\[ \cos^{-1}(0) = \| \text{ the angle in } [0, \pi/2] \text{ whose cosine is } \]
\[
\frac{\pi}{2} < x < 3\pi \quad \text{or} \quad \frac{3\pi}{2} < x < 2\pi
\]

\[
0 \leq \text{tan}x < \infty\]
\[
\frac{\cos(x)}{y'} = \frac{\cos(y)}{1} = \frac{1}{\cos(y)} \cdot y' = 1
\]

Implication:

Can differentiate:

Since \( \sin(y) = x \),

\[
x = \sin(y) = x
\]
\[ y = \frac{5}{x-1} \]

\[ y = \sin x - 1(x) \]

\[ \sin y + \cos y = 1 \]

\[ \cos y = 1 - \sin^2 y \]

\[ \sqrt{1 - x^2} \]

\[ y = 2 \]

\[ y = \frac{5}{x-1} \]