THEOREM 8.14 The Ratio Test

Let \( \sum a_k \) be an infinite series with positive terms and let
\[
    r = \lim_{k \to \infty} \frac{a_{k+1}}{a_k}.
\]

1. If \( 0 \leq r < 1 \), the series converges.
2. If \( r > 1 \) (including \( r = \infty \)), the series diverges.
3. If \( r = 1 \), the test is inconclusive.
Example: According to the Ratio Test, is the following series convergent, divergent, or is the Ratio Test inconclusive?

(a) \[ \sum_{k=1}^{\infty} \left( \frac{4^k}{k \cdot 3^k} \right) \]
Example: According to the Ratio Test, is the following series convergent, divergent, or is the Ratio Test inconclusive?

(b) \[ \sum_{k=1}^{\infty} \frac{2^k}{k!} \]

First: Recall that \( k! = \)

So \( \frac{(k+1)!}{k!} = \)
So … Ratio Test. Convergent, divergent, or inconclusive?

\[ \sum_{k=1}^{\infty} \left( \frac{2^k}{k!} \right) \]
Example: According to the Ratio Test, is the following series convergent, divergent, or is the Ratio Test inconclusive?

\[(c) \sum_{k=1}^{\infty} \left( \frac{k + 2}{2k + 9} \right) \]
THEOREM 8.15   The Root Test

Let \( \sum a_k \) be an infinite series with nonnegative terms and let

\[
\rho = \lim_{k \to \infty} k \sqrt[k]{a_k}.
\]

1. If \( 0 \leq \rho < 1 \), the series converges.
2. If \( \rho > 1 \) (including \( \rho = \infty \)), the series diverges.
3. If \( \rho = 1 \), the test is inconclusive.
Example: Is the following series convergent or divergent?

(d) \[ \sum_{k=0}^{\infty} \left( \frac{5k + 3k^3}{7k^3 + 2} \right)^k \]
Example: Is the following series convergent or divergent?

\[
e^{-3e} + \frac{9}{e^2} + \frac{27}{e^3} + \frac{81}{e^4} + \ldots
\]