**Study Questions for Exam One**

Exam One will cover material from Chapter 1, Chapter 2, and Section 7.5. This is the material covered in Homework Assignments 1 through 4. You will *not* be tested on any sections or topics that we have skipped, such as Section 1.1, Figurate Numbers in Section 1.2, etc.

Note that this is a study guide, not a sample exam - it is much longer than your exam will be. However, the ideas and the question types represented here (along with your written homework and online homework) will help prepare you for your exam.

Solutions to these questions can be found at:

http://www.math.wsu.edu/faculty/martin/Math105/ex1SGsoln.pdf

On the exam, you **must** show your work for all problems. Credit will not be given for correct answers that are not justified. Keep that in mind as you work through these questions.

The following information and formulas (and no others) will be provided on your exam in exactly this format. Knowing the other important formulas not listed here, and knowing when and how to use them, is up to you.

**Possibly useful information:**

General term ($n^{th}$ term) of an arithmetic sequence: \( a_n = a_1 + (n - 1)d \)

General term ($n^{th}$ term) of a geometric sequence: \( a_n = a_1 \times r^{n-1} \)

1. An auditorium has a section where the seating can be arranged so the first row has 11 seats, the second row has 15 seats, the third row has 19 seats and so on. There are 30 rows in the section.

   (a) What type of sequence is represented by the number of seats in each row?

   (b) Give the $n^{th}$ term (the general term) for the sequence.

   (c) How many seats are in the last row of the section?

2. A hot tub is filled with water at a temperature of 75º F. The temperature increases by 10% each hour.

   (a) What type of sequence is represented by the temperature of the water each hour?

   (b) Give the $n^{th}$ term (the general term) for the sequence.

   (c) What will the temperature of the hot tub be after 3 hours, to the nearest tenth of a degree?
3. For each sequence, determine if it is an arithmetic sequence, a geometric sequence, or neither. Give the next two terms in the sequence. If the sequence is arithmetic or geometric, write a formula for the general term (the \( n \)th term). If it is neither arithmetic nor geometric, indicate that the general term does not apply.

(a) 3, 8, 13, 18, ...
(b) 5, -1, 4, 3, 7, ...
(c) 4, 10, 25, 62.5, ...
(d) 105, 100, 95, 90, ...
(e) 5, 7, 11, 17, 25, ...

4. Determine the indicated term in the given sequence.

(a) The 12th term of 4, 11, 18, ...
(b) The 12th term of 3, 9, 27, ...

5. Solve the following problems using Polya's Four-Step Method. Explain your solution as a step-by-step process, listing each of the four steps in the appropriate place and describing briefly how you applied each step to the problem. Answer using complete sentences.

(a) A pet shop has a total of 18 dogs and birds. Altogether there are 52 feet. How many dogs are there and how many birds?

(b) The flower peddler has red flowers with five petals each and white flowers with eight petals each. He has a total of 9 flowers with a total of 54 petals. How many red flowers are there and how many white flowers?

(c) How many different ways can you make change for a 50-cent coin using nickels and dimes?

(d) How many posts does it take to support a straight free-standing fence 210 feet long if posts are placed every 10 feet?

(e) How many cuts are needed to divide an 80-foot log into 1-foot sections?

(f) What is the smallest number of people you would need to have in a room to guarantee that at least three of them were born in the same month?
6. List all of the elements of each set. Use set notation and the listing method to describe the set.
   
   (a) The set $A$ of counting numbers less than seven.
   
   (b) The set $B$ of letters in the word “bamboo.”
   
   (c) $C = \{x|x$ is a counting number less than zero$\}$
   
   (d) $D = \{x|x$ is an integer between 3 and 7$\}$
   
   (e) $E = \{x|x$ is a whole number smaller than 10$\}$
   
   (f) $F = \{x|x$ is an integer smaller than 10$\}$

7. Denote each set by set-builder notation, using $x$ as the variable.
   
   (a) The set $A$ of counting numbers less than seven.
   
   (b) The set $B$ of letters in the word “bamboo.”
   
   (c) $C = \{-3, -2, -1, 0, 1, \ldots\}$

8. Indicate if the following statements are true or false. Explain briefly.
   
   (a) $0 \not\in \{1, 2, 3, 4, \ldots\}$
   
   (b) $12 \in \{x|x$ is an even number$\}$
   
   (c) $\pi \in \mathbb{Q}$
   
   (d) $\pi \in \mathbb{R}$

9. Find the cardinal number of each set.
   
   (a) The set $A$ of counting numbers between three and eight.
   
   (b) $B = \{-1, 0, 1, 2, 3, \ldots, 99, 100\}$
   
   (c) $C = \{x|x$ is a month in the year$\}$
   
   (d) $M = \{m, o, n, t, a, n, a\}$
   
   (e) $P$, if $n(P \cap Q) = 15$, $n(P \cup Q) = 50$, and $n(Q) = 20$.

10. State whether the sets in each pair are equal. Explain your answer.
    
    (a) $\{1, 2, 3, 4\}$ and $\{x|x$ is a whole number between one and four$\}$
    
    (b) $\{1/10, 2/10, 3/10\}$ and $\{0.1, 0.2, 0.3\}$
    
    (c) $\mathbb{N}$ and $\{x|x$ is a whole number greater than zero$\}$

11. Let $U = \{1, 2, 3, 4, 5, a, b, c, d, e\}$, $A = \{1, 2, 3, 4\}$, $B = \{2, 4\}$, and $C = \{a, b, c, d\}$.
    
    (a) Use a Venn diagram to illustrate sets $A$, $B$, $C$, and $U$.
    
    (b) Is the statement $A \subseteq B$ true or false? Explain.
    
    (c) Is the statement $B \subseteq A$ true or false? Explain.
    
    (d) List the elements in $A'$.
    
    (e) List the elements in $U'$. 
12. Let \( U = \{1, 2, 4, 5, a, b, c, d, e\} \). Find the complement of the set.
   
   (a) \( A = \{2, 4, b, d\} \)
   
   (b) \( S = \emptyset \)
   
   (c) \( N = \{a\} \)

13. Let \( U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \), \( A = \{2, 3, 4, 5, 6\} \), \( B = \{2, 4, 6, 8, 10\} \), \( C = \{3, 6, 9\} \).

   (a) Place the set elements in the proper location on the Venn diagram.

   (b) List the elements in the following sets:

   \[ A \cap B = \]
   
   \[ A \cup B = \]
   
   \[ B' = \]
   
   \[ C - B = \]
   
   \[ B - C = \]
   
   \[ B - C' = \]
   
   \[ (B - C)' = \]

14. The lists below show five agricultural crops in Alabama, Arkansas, and Louisiana.

<table>
<thead>
<tr>
<th>Alabama</th>
<th>Arkansas</th>
<th>Louisiana</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soybeans (S)</td>
<td>Soybeans (S)</td>
<td>Soybeans (S)</td>
</tr>
<tr>
<td>Peanuts (P)</td>
<td>Rice (R)</td>
<td>Sugarcane (N)</td>
</tr>
<tr>
<td>Corn (C)</td>
<td>Cotton (T)</td>
<td>Rice (R)</td>
</tr>
<tr>
<td>Hay (H)</td>
<td>Hay (H)</td>
<td>Corn (C)</td>
</tr>
<tr>
<td>Wheat (W)</td>
<td>Wheat (W)</td>
<td>Cotton (T)</td>
</tr>
</tbody>
</table>

Let \( U \) be the smallest possible universal set that includes all of the crops listed, and let \( A, K \) and \( L \) be the sets of the five crops in Alabama, Arkansas, and Louisiana, respectively. Find each of the following sets.

(a) \( A \cap K \cap L \)
(b) \( A \cup K \cup L \)
(c) \( U \)
(d) \((A \cap K \cap L)'\)
(e) \( A' \)
(f) \( A - K \)

15. Use a 3-loop Venn diagram to shade the regions representing the given sets. Use color and/or directional shading as needed to show intermediate steps, but your final answer should be shaded in just one color.

(a) \( C' \)  
(b) \( A \cup B \)  
(c) \( C' \cap (A \cup B) \)  
(d) \( C' \cup (A \cup B) \)

16. Describe the conditions under which the statement is true.

(a) \( A \cap B = A \)
(b) \( A \cup \emptyset = U \)

17. Out of forty students, 14 are taking English Composition and 29 are taking Chemistry. If five students are in both classes, how many students are in neither class? How many are in Chemistry only?

18. A survey of 260 families showed that:

99 families had a dog;
76 families had a cat;
34 families had a dog and a cat;
10 families had a dog and a parakeet but not a cat;
10 families had a cat and a parakeet;
98 families had neither a cat nor a dog nor a parakeet;
8 families had a cat and dog and a parakeet.

(a) Draw and label a Venn diagram to illustrate this situation. Fill in each section with the appropriate numbers for each set.
(b) How many families had a parakeet only?
(c) How many families had a dog and a cat but not a parakeet?
19. The numbers in the Venn Diagram below represent cardinalities.

![Venn Diagram with numbers](image)

(a) Find $n(A' \cap B' \cap C)$.
(b) Find $n(B \cup C)$.

20. Given:

\[
\begin{align*}
    n(A) &= 80 \\
    n(B) &= 88 \\
    n(C) &= 82 \\
    n(A \cap B) &= 16 \\
    n(A \cap C) &= 18 \\
    n(B \cap C) &= 12 \\
    n(A \cap B \cap C) &= 10 \\
    n(A' \cap B' \cap C') &= 161
\end{align*}
\]

Find $n(U)$.

21. Find the cardinal number of the indicated set.

(a) If $n(A) = 7$, $n(B) = 15$ and $n(A \cap B) = 5$, what is $n(A \cup B)$?
(b) If $n(B) = 60$, $n(A \cap B) = 11$, and $n(A \cup B) = 105$, find $n(A)$.

22. Convert each number from standard notation to scientific notation.

(a) 6209
(b) 0.00047

23. Convert each number from scientific notation to standard notation.

(a) $2.28 \times 10^3$
(b) $1.4 \times 10^{-7}$

24. Use scientific notation to perform the computation. Leave the answer in scientific notation.

(a) $(1.5 \times 10^7)(0.2 \times 10^{-4})$
(b) \[ \frac{2 \times 10^{-4}}{8 \times 10^2} \]

(c) The national debt of a small country is $6,900,000,000 and the population is 2,560,000. What is the amount of debt per person? Give the answer in scientific notation correct to two decimal places.