1. (16pts) Three married couples are seated together at the counter at Monty’s Blue Plate Diner, occupying six consecutive seats. How many arrangements are there with no wife sitting next to her own husband? We do NOT require men and women to alternate. For full credit, you must use the inclusion-exclusion method. Write your answer in a single number.

\[ A_i - \text{ith couple sit together} \quad (i = 1, 2, 3) \]

\[ |A_i| = 2 \cdot 5! \] (treat this couple as 1 person, husband & wife may switch)

Similarly, \( |A_i \cap A_j| = 2 \cdot 4! \), \( |A_i \cap A_j \cap A_k| = (2)(2)(2)(3)! \)

\[ |A_i \cap A_j \cap A_k| = 6! - (3\cdot 2 \cdot 5! + (\begin{array}{c} 3 \\ 2 \end{array})(2)(2)(4!) - (3)(3)! \]

\[ = 6! - 6! + 3 \cdot 4 \cdot 24 - 8 \cdot 6 = 240 \]

2. (12pts) Determine the number of permutations of \( \{1, 2, \ldots, 7\} \) in which at least two integer are in their natural positions. Write your answer in an expression.

\[ \binom{7}{2} 7! - \binom{2}{2} 7! - \binom{7}{1} D_7 \]

\( \text{no int.} \quad \text{exactly one int.} \)

\( \text{in natural position} \quad \text{in natural position} \)
3. (16pts) Count the permutations \( i_1, i_2, i_3, i_4, i_5, i_6 \) of \( \{1, 2, 3, 4, 5, 6\} \), where \( i_1 \neq 1, 2, i_3 \neq 3, \text{ and } i_6 \neq 5, 6 \). Write your answer in a single number.

\[
\begin{align*}
R_1 &= 8 \times \left( \begin{array}{c} 4 \\
\text{both } \in S_1 \setminus 1 \\
\text{one } \in S_2 \setminus 1 \\
\end{array} \right) \\
&> 2^2 \\
R_2 &= 22 \times \left( \begin{array}{c} 2 \\
2 \in S_1 \setminus 1 \\
1 \in S_2 \setminus 1 \\
\end{array} \right) \\
&= 22 \times 8 \\
R_3 &= 25 \times \left( \begin{array}{c} 1 \\
1 \in S_1 \setminus 2 \\
1 \in S_2 \setminus 2 \\
\end{array} \right) \\
&= 25 \\
R_4 &= 12 \times \left( \begin{array}{c} 4 \\
3 \in S_1 \setminus 2 \\
2 \in S_2 \setminus 2 \\
\end{array} \right) \\
&= 12 \times 4 \\
R_5 &= 1 \times 2 = 2 \\
&\quad (3 \in S_1 \setminus 2 \in S_2)
\end{align*}
\]

\[
6! - 8(5!) + 22(4!) - 25(3!) + 12(2!) - 2(1!) = (-2)(5!) + 3!(88 - 25) + 22 = \boxed{160}
\]

4. (10pts) A class of ten boys are seated in a row in a classroom. In order that a child not have the same person to his left except the leftmost boy, on the second day the boys decide to switch positions so that no boy except the leftmost boy has the same boy to his left as the first day. In how many ways can they switch positions? Write your answer in an expression.

\( \boxed{10} \)
5. (12pts) For \( n > 0 \), let \( a_n \) be the number of ways to tile a 1-by-\( n \) strip with 1-by-2 tiles and 1-by-3 tiles (so that \( a_0 = 1, a_1 = 0, \) and \( a_2 = 1 \)). Find a recurrence relation satisfied by \( a_n \). You don't need to solve this recurrence relation.

2 ways to tile:

Start with 1-by-2:

\[ a_{n-2} \]

Start with 1-by-3:

\[ a_{n-3} \]

So

\[ a_n = a_{n-2} + a_{n-3} \]

6. (10pts) Murial is packing a lunch with seven items, including at least one apple, an even number of bananas, up to 2 candy bars, and an odd number of dates. The number of ways she can do this is given by the coefficient of \( x^7 \) in the expansion of

\[ g(x) = (x + x^2 + \cdots + x^7)(1 + x^2 + x^4 + x^6)(1 + x + x^2)\left(x + x^3 + x^5 + x^7\right)\]
7. (10pts) Mike is laying a row a 10 square tiles that come in the colors red, green, blue, and yellow. He wants to include any number of red, a positive even number of green, an odd number of blue, and at most two yellow tiles. The number of ways he can lay the 10 tiles is given by the coefficient of \( \frac{x^{10}}{10!} \) in the expansion of
\[
g^{(10)}(x) = (1 + x + \frac{x^2}{2!} + \ldots + \frac{x^{10}}{10!})(1 + \frac{x^4}{4!} + \ldots + \frac{x^{10}}{10!})
\]
\[
= (1 + \frac{x^3}{3!} + \ldots + \frac{x^{9}}{9!})(1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots + \frac{x^{10}}{10!})
\]

8. (14pts) Solve the recurrence relation \( h_n = 2h_{n-1} + h_{n-2} - 2h_{n-3}, \ (n \geq 3) \) with initial values \( h_0 = 1, h_1 = 2, \) and \( h_2 = 0. \)

Characteristic equ. \( x^3 - 2x^2 - x + 2 = 0 \)
\[
x(x-2)(x-1) = 0
\]
\[
(x-2) (x^2-1) = 0
\]
\[
(x-2) (x+1)(x-1) = 0, \quad q \rightarrow 2, -1
\]

So \( h_n = c_1 (2^n) + c_2 (1^n) + c_3 (-1)^n \)

\( h_0 = 1 : \quad c_1 + c_2 + c_3 = 1 \)
\( h_1 = 2 : \quad 2c_1 + c_2 - c_3 = 2 \)
\( h_2 = 0 : \quad 4c_1 + c_2 + c_3 = 0 \)

Solve by elimination:
\[
c_1 = -\frac{1}{3}, \quad c_2 = 2, \quad c_3 = -\frac{2}{3}
\]

So \( h_n = -\frac{1}{3} (2^n) + 2 - \frac{2}{3} (-1)^n \)