Math 201 Practice Test 3, Fall 2014
(4-6, 5-1, 5-2, 5-3, 6-2, 7-3, 7-4)

Note: This should not be regarded as an altered version of your exam. It's just for practice.

1. Given the system of linear equations:
   \[ x + y = 15 \]
   \[ 2x - 3y = 10 \]
   a. Write it as a matrix equation:
   \[
   \begin{bmatrix}
   1 & 1 \\
   2 & -3 \\
   \end{bmatrix}
   \begin{bmatrix}
   x \\
   y \\
   \end{bmatrix}
   =
   \begin{bmatrix}
   15 \\
   10 \\
   \end{bmatrix} = B
   \]

   b. Find the inverse of the coefficient matrix:
   \[
   \begin{bmatrix}
   1 & 1 & 0 \\
   2 & -3 & 1 \\
   \end{bmatrix} \rightarrow
   \begin{bmatrix}
   1 & 1 & 0 \\
   0 & -5 & 2 \\
   \end{bmatrix} \rightarrow
   \begin{bmatrix}
   1 & 1 & 0 \\
   0 & -5 & 2 \\
   \end{bmatrix} \rightarrow
   \begin{bmatrix}
   1 & 0 & 2/5 \\
   0 & 1 & -1/5 \\
   \end{bmatrix}
   \]
   \[
   A^{-1} = \begin{bmatrix}
   3/5 & 1/5 \\
   2/5 & -1/5 \\
   \end{bmatrix}
   \]

   c. Use the inverse matrix from part b) to solve the system of linear equations.
   \[
   X = A^{-1}B = \begin{bmatrix}
   3/5 & 1/5 \\
   2/5 & -1/5 \\
   \end{bmatrix} \begin{bmatrix}
   15 \\
   10 \\
   \end{bmatrix} = \begin{bmatrix}
   11 \\
   4 \\
   \end{bmatrix}, \quad \begin{bmatrix}
   x_1 = 11 \\
   x_2 = 4 \\
   \end{bmatrix}
   \]
2. Solve: $-3x + y \leq 6$
$2x - 3y > -6$
3. For the system of inequalities:

\[ \begin{align*}
2x + y &\leq 10 \\
3x + y &\geq 6 \\
 x, y &\geq 0
\end{align*} \]

a) Solve graphically;

b) Is the region bounded or unbounded? **bounded**

c) Find the coordinates of all of the corner points.

\[(2, 0), (5, 0), (0, 6), (0, 10)\]
4. Draw the solution region (feasible region), then determine whether the region is bounded or unbounded or empty. Calculate the coordinates of all corner points if there is any.

\[ 2x + 3y \geq 12 \]

a. \[ -x + 3y \geq 3 \]
\[ 0 \leq y \leq 3 \]

\[ \begin{align*}
A: & \quad -x + 3y = 3 \\
2x + 3y &= 12 \\
\frac{2x + 3y = 12}{3x} &= 9 \\
3y &= 6 \\
x &= 3 \\
y &= 2 \\
\end{align*} \]

\[ A: (3, 2) \]

\[ B: \quad \begin{align*}
y &= 3 \quad \text{and} \\
-x + 3y &= 3 \\
-x + 3(3) &= 3 \\
x &= -6, \quad x = 6 \\
\end{align*} \]

\[ B: (6, 3) \]

\[ C: \quad \begin{align*}
y &= 3 \quad \text{and} \\
2x + 3(y) &= 12 \\
2x + 9 &= 12 \\
x &= 3/2 \\
\end{align*} \]

\[ C: \left(\frac{3}{2}, 3\right) \]

**Empty FR**
5. Maximize and minimize $P = 7x + 2y$ subject to

$$\begin{align*}
3x + y &\leq 50 \\
5x + y &\leq 70 \\
2x + y &\leq 80 \\
x, y &\geq 0
\end{align*}$$

a. Find the feasible region;

b. Find the coordinates of all corner points;

$A: \begin{cases}
3x + y = 50 \\
5x + y = 70 \\
2x = 20 \Rightarrow x = 10
\end{cases}$

$3(10) + y = 50 \Rightarrow y = 20$, $A: (10, 20)$

$(0, 0), (10, 20), (14, 0), (0, 50)$

c. Find the maximum and minimum value if there is any; indicate the coordinates of the points where the maximum or minimum occurs.

$$P = 7x + 2y$$

$(0, 0)$

$P(10, 20) = 110$  \(\text{Max } P = 110\) \(\boxed{(10, 20)}\)$

$P(14, 0) = 98$  \(\boxed{(14, 0)}\)$

$P(0, 50) = 100$  \(\boxed{(0, 50)}\)$

$\text{Min } P = 0$  \(\boxed{(0, 0)}\)$
6. Maximize and minimize \( C = 2x + 2y \) subject to

\[
\begin{align*}
3x + 2y &\leq 160 \\
x + 2y &\leq 80 \\
5x + 2y &\leq 200 \\
x, y &\geq 0
\end{align*}
\]

a. Find the feasible region:

b. Find the coordinates of all corner points:

A: \( (3x + 2y = 160) \)

\[
\begin{align*}
3x + 2y &= 160 \\
2x &= 40, \quad x = 20 \\
2y &= 80, \quad y = 40, \\
(20, 50)
\end{align*}
\]

B: \( (x + 2y = 80) \)

\[
\begin{align*}
5x + 2y &= 200 \\
4x + 2y &= 80 \\
x &= 40, \\
y &= 20, \\
(40, 20)
\end{align*}
\]

(80, 0) \( \quad \) (0, 100)

(c. Find the maximum and minimum value if there is any; indicate the coordinates of the points where the maximum or minimum occurs.

\[
\begin{align*}
C &= 2x + 2y \\
(20, 50) : \quad C &= 2(20) + 2(50) = 140 \\
(40, 20) : \quad C &= 2(40) + 2(20) = 120 \\
(80, 0) : \quad C &= 2(80) + 2(0) = 160 \\
(0, 100) : \quad C &= 2(0) + 2(100) = 200
\end{align*}
\]

\( \min C = 120 \) \( \quad \) (40, 20)

\( \max C \) (unbounded)
7. Given the linear programming problem:

\[ 2x_1 + x_2 \leq 8 \]

Maximize \( P = 6x_1 + 2x_2 \) subject to \( x_1 + 2x_2 \leq 10 \) using \textbf{Simplex Method}.

\[ x_1, x_2 \geq 0 \]

\[
\begin{align*}
2x_1 + x_2 + S_1 & = 8 \\
x_1 + 2x_2 + S_2 & = 10 \\
-6x_1 - 2x_2 + P & = 0
\end{align*}
\]

\[ x_1 \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & 4 \end{bmatrix} - R_2 \] 
\[ 6R_1 \rightarrow R_3 \]

\[ x_1 \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 & 4 \end{bmatrix} \]

\[ x_2 \begin{bmatrix} 0 & 3\frac{1}{2} & -\frac{1}{2} & 1 & 0 & 6 \end{bmatrix} \]

\[ P \begin{bmatrix} 0 & -1 & -3 & 6 & -1 & -24 \end{bmatrix} \]

No negatives, done.

\[ \text{Max } P = 24 \text{ as } x_1 = 4, \ x_2 = 0 \]
8. (manufacturing) A company manufactures outdoor furniture consisting of regular chairs, rocking chairs, and chaise lounges. Each piece of furniture passes through three different production departments: fabrication, assembly and finishing. Each regular chair takes 1 hour to fabricate, 2 hours to assemble, and 3 hours to finish. Each rocking chair takes 2 hours to fabricate, 2 hours to assemble, and 3 hours to finish. Each chaise lounge takes 3 hours to fabricate, 4 hours to assemble, and 2 hours to finish. There are 2,500 labor-hours available in the fabrication department, 3000 in the assembly department, and 3,500 in the finishing department. The company makes a profit $17 on each regular chair, $24 on each rocking chair, and $31 on each chaise lounge. How many chairs of each type should the company produce in order to maximize profit? What is the maximum profit?

a. Assign variables to unknowns and write the objective function;

\[ x_1 \text{ --- # regular chairs} \]
\[ x_2 \text{ --- # rocking chairs} \]
\[ x_3 \text{ --- # chaise lounges} \]

\[ \text{Profit} \ P = 17x_1 + 24x_2 + 31x_3 \]

b. Set up the constraints;

\[ \text{fabrication:} \quad x_1 + 2x_2 + 3x_3 \leq 2500 \]
\[ \text{assembly:} \quad 2x_1 + 2x_2 + 4x_3 \leq 3000 \]
\[ \text{finishing:} \quad 3x_1 + 3x_2 + 2x_3 \leq 3500 \]
\[ x_1, x_2, x_3 \geq 0 \]
9. Solve the following linear programming by the simplex method.

\[-x_1 + x_2 \leq 2\]
\[-x_1 + 3x_2 \leq 12\]
Maximize \( P = -x_1 + 2x_2 \) subject to
\[x_1 - 4x_2 \leq 4\]
\( x_1, x_2 \geq 0 \)

\[-x_1 + x_2 + s_1 = 2\]
\[-x_1 + 3x_2 + s_2 = 12\]
\[x_1 - 4x_2 + s_3 = 4\]
\[x_1 - 2x_2 + \mu = 0\]

\[
\begin{bmatrix}
-x_1 & x_2 & s_1 & s_2 & s_3 & \mu & P \\
1 & -1 & 1 & 0 & 0 & 0 & 2 \\
-1 & 3 & 0 & 1 & 0 & 0 & 12 \\
1 & -4 & 0 & 0 & 1 & 0 & 4 \\
1 & 0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
\]

\[
\begin{align*}
2/1 &= 2 \\
12/3 &= 4 \\
\end{align*}
\]

-3\(R_1+R_2 \rightarrow R_2\)
4\(R_1+R_3 \rightarrow R_3\)
2\(R_1+R_4 \rightarrow R_4\)

\[
\begin{bmatrix}
-x_1 & 1 & 0 & 0 & 0 & 2 \\
0 & 1 & 0 & 0 & 0 & 6 \\
0 & -3 & 1 & 0 & 0 & 12 \\
-1 & 0 & 2 & 0 & 0 & 4
\end{bmatrix}
\]

\[
\begin{align*}
\text{No} \\
\text{No} \\
\end{align*}
\]

\[
\begin{bmatrix}
-x_1 & 1 & 0 & 0 & 0 & 2 \\
0 & 1 & 0 & 0 & 0 & 3 \\
0 & -3 & 1 & 0 & 0 & 12 \\
-1 & 0 & 2 & 0 & 0 & 4
\end{bmatrix}
\]

\[
\begin{align*}
\text{No} \\
\text{No} \\
\end{align*}
\]

\[
\begin{bmatrix}
-x_1 & 1 & 0 & 0 & 0 & 2 \\
0 & 1 & 0 & 0 & 0 & 3 \\
0 & 0 & 4 & 0 & 1 & 12 \\
-1 & 0 & 2 & 0 & 0 & 4
\end{bmatrix}
\]
\[
\begin{array}{ccccccc}
X_1 & X_2 & S_1 & S_2 & S_3 & P \\
\begin{array}{cccccc}
R_2 + R_1 \rightarrow R_1 \\
R_2 + R_4 \rightarrow R_4 \\
3R_2 + R_3 \rightarrow R_3 \\
\end{array}
\end{array}
\]

\[
X_2 \left[ \begin{array}{cccc}
0 & 1 & -1 & 1 & 0 & 0 & 5 \\
0 & 1 & -1 & 1 & 0 & 0 & 3 \\
0 & 1 & -1 & 3 & 1 & 0 & 21 \\
0 & 0 & 1 & 1 & 0 & 1 & 7 \\
\end{array} \right]
\]

\[\text{Max } P = 7 \text{ s.t. } X_1 = 3, \text{ } X_2 = 5\]
10. 100 shoppers were asked if they owned a dog or a cat. The survey found that 40 people owned a cat, 42 people owned a dog, and 28 people owned neither a cat nor a dog. How many people in the survey owned: (hint: use a Venn Diagram)

a) a dog and a cat?

\[(40 + 42 - x) + 28 = 100 \]
\[82 - x + 28 = 100 \]
\[110 - x = 100 \]
\[x = 10 \]

b) a dog but not a cat?

\[42 - 10 = 32 \]

11. How many 3-letter code words are possible using the eight letters in the word ‘grateful’ if:

a. No letter can be repeated?

\[8 \times 7 \times 6 = 336 \]

b. Letters can be repeated?

\[8 \times 8 \times 8 = 512 \]

c. Adjacent letters cannot be alike?

\[8 \times 7 \times 7 = 392 \]
12. Solve the following problems using $P_{n,r}$ or $C_{n,r}$:

a. How many 4-digit opening combinations are possible on a combination lock with 6 digits if the digits cannot be repeated?

\[ P_{6,4} = \frac{6!}{2!} = \frac{6 \times 5 \times 4 \times 3 \times 2!}{2!} = 360 \]

b. Six tennis players have made the finals. If each of the 6 players is to play every other player exactly once, how many games must be scheduled?

\[ C_{6,2} = \frac{6!}{2!4!} = \frac{3 \times 6 \times 5 \times 4!}{2 \times 4!} = 15 \]

13. A software development department consists of 7 women and 4 men.

A) How many ways can the department select a chief programmer, a backup programmer, and a programming librarian?

\[ P_{11,3} = \frac{11!}{8!} = \frac{11 \times 10 \times 9 \times 8!}{8!} = 990 \]

B) How many of the selections in part A) consist entirely of women?

\[ P_{7,3} = \frac{7!}{4!} = \frac{7 \times 6 \times 5 \times 4!}{4!} = 210 \]

C) How many ways can the department select a team of 3 programmers to work on a particular project?

\[ C_{11,3} = \frac{11!}{3!8!} = \frac{11 \times 10 \times 9 \times 8!}{3 \times 2 \times 1 \times 8!} = 165 \]