Math 201 Practice Final, Fall 2010

Note: this is for practice only. It should not be regarded as an altered version of the final.

1. Solving for the indicated variable.
   (a) \( F = \frac{1}{3} C + 16 \), for \( C \).

   (b) \( Ax + By + Cz = G \), for \( z \).

2. Write line equations for the following situations.
   (a) A horizontal line that passes through \((x_0; y_0)\).

   (b) A line that passes through \((20, -30)\) and \((-100, 30)\).

3. Find and simplify \( \frac{f(x+h) - f(x)}{h} \), assuming \( h \neq 0 \) given \( f(x) = 2x^2 - 3x + 5 \).
4. Determine the domain of each function.

   a. \( f(x) = \frac{x-1}{x+1} \)

   b. \( g(x) = \sqrt{7-x} \)

   c. \( f(x) = \frac{x-1}{\sqrt{x+1}} \)

5. Graph \( f(x) = \begin{cases} x^2, & x < 0 \\ x+3, & x \geq 0 \end{cases} \)

6. Given \( f(x) = -(x-2)^2 + 4 \), find the intercepts, vertex, maximum or minimum, and range.
7. Determine the vertex form of the quadratic function \( f(x) = x^2 - 8x + 13 \). Find the intercepts, maximum or minimum, and range.

8. Given the polynomial \( f(x) = -\left( x^2 + 4 \right) \left( x^2 - 1 \right)(4x - 1) \), determine the degree and all intercepts.

9. For the function \( f(x) = \frac{x+1}{x-2} \), determine the intercepts, domain, and find any asymptotes. Graph the function and label all intercepts and asymptotes.

Intercepts: __________________________

Asymptotes: __________________________

Domain: __________________________
10. Solve: \( 4^{5x-x^2} = 4^{-6} \).

11. Solve \( \log_{10} x = \frac{3}{2} \log_{10} 4 - \frac{2}{3} \log_{10} 8 + 2 \log_{10} 2 \).

12. Solve: \( \log_{10} (x - 1) - \log_{10} (x + 1) = 1 \).

13. Express as a single matrix: \( \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2 \\ 7 \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \end{bmatrix} \begin{bmatrix} 2 & -4 \\ -2 & -3 \end{bmatrix} \)
14. Solve using inverses

\[
\begin{bmatrix}
1 & 1 & 0 \\
2 & 3 & -1 \\
1 & 0 & 2
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix} =
\begin{bmatrix}
2 \\
0 \\
4
\end{bmatrix}
\]
15. A supplier manufactures car and truck frames at two different plants. The production rates (in frames per hour) for each plant are given in the table:

<table>
<thead>
<tr>
<th>Plant</th>
<th>Car Frames</th>
<th>Truck Frames</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>B</td>
<td>8</td>
<td>8</td>
</tr>
</tbody>
</table>

How many hours should each plant be scheduled to operate to exactly fill each of the orders in the following table? Set up the linear system only. Do not solve.

<table>
<thead>
<tr>
<th></th>
<th>Order 1</th>
<th>Order 2</th>
<th>Order 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Car Frames</td>
<td>3000</td>
<td>2800</td>
<td>2600</td>
</tr>
<tr>
<td>Truck Frames</td>
<td>1600</td>
<td>2000</td>
<td>2200</td>
</tr>
</tbody>
</table>

16. A person has $5000 to invest, part at 5% and the rest at 10%. How much should be invested at each rate to yield $400 per year? Solve using augmented matrix methods.
17. Sketch the feasible region of the problem below, labeling the graph appropriately and solve, using the method of linear programming.

Minimize $Z = 5x + 6y$

Subject to: 

$.10x + 20y \geq 300$

$20x + 5y \geq 250$

$x, y \geq 0$
18. Summarize the information in the problem below in the given table, assign variables to all unknowns, identify the objective function and all constraints. *Do not solve.*

A candy manufacturer has 1000 pounds of chocolate, 200 pounds of nuts and 100 pounds of dried fruit in stock. The Special Mix requires 3 pounds of chocolate and 1 pound each of nuts and fruit and brings in $10 per box in profit. The Regular Mix requires 4 pounds of chocolate, \( \frac{1}{2} \) pound of nuts and no fruit and brings in $6 per box in profit. The Purist Mix requires 5 pounds of chocolate, no nuts or fruit and brings in $4 per box in profit. How many boxes of each type should be produced to maximize profit?

<table>
<thead>
<tr>
<th></th>
<th>Boxes Special</th>
<th>Boxes Regular</th>
<th>Boxes Purist</th>
<th>Total Available</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chocolate</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nuts</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fruit</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Precisely define the variables to be used in your constraints:

Let \( x = \) __________________

Let \( y = \) __________________

Let \( z = \) __________________

**Objective Function:** __________________

**Constraints.**
19. A company manufactures two products, A and B. Each product requires sanding and finishing according to the table below:

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sanding</td>
<td>2 hr.</td>
<td>3 hr.</td>
</tr>
<tr>
<td>Finishing</td>
<td>2 hr.</td>
<td>1 hr.</td>
</tr>
</tbody>
</table>

Twelve hours of sanding time and 8 hours of finishing time are available per week. The profit per unit on products A and B is $5 and $10 respectively.

Represent unknowns with variables:

\[ x = \]  

\[ y = \]  

State the objective function (Profit):

Identify all constraints:

Graph the feasible region: (on next page)
Identify coordinates of all corners of the feasible region:

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Process to Maximize Profit:

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Result: ______
20. Use simplex method to solve the problem.
(a) 
maximize: \( P = 4x_1 - 3x_2 + 2x_3 \)
subject to: 
\[ x_1 + 2x_2 + 3x_3 \leq 5 \]
\[ 3x_1 + 2x_2 + 2x_3 \leq 22 \]
\[ x_1, x_2, x_3 \geq 0 \]
(b) 
maximize: \( P = 2x_1 + 3x_2 - 3x_3 \) 
subject to: 
\( x_1 - x_2 - 2x_3 \leq 3 \) 
\( x_1 + x_2 \leq 5 \) 
\( x_1, x_2, x_3 \geq 0 \)
21. In how many ways can three people be seated in a row of 7 chairs?

22. In drawing 5 cards from a 52-card deck without replacement, (you may write your answer in terms of factorials.)
   a. How many hands contain 4 hearts and 1 spade?

   b. What is the probability of getting 4 hearts and 1 spade?

   c. If the probability of getting a pair is 1/247, what are the odds of getting a pair?

   d. If the odds against getting two pairs are 597 to 2, what is the probability of getting two pairs?

23. If a coin is tossed 4 times in a row, what is the probability of getting at least one tail? What are the odds against getting at least one tail?
24. (2pts each part) How many 3-letter code words are possible using the first 8 letters of the alphabet if:

a. No letter can be repeated?

b. Letters can be repeated?

c. Adjacent letters cannot be alike?

25. (3pts each part) Solve the following problems using \( P_n \) or \( C_n \):

a. How many 3-digit opening combinations are possible on a combination lock with 6 digits if the digits cannot be repeated?

b. Five tennis players have made the finals. If each of the 5 players is to play every other player exactly once, how many games must be scheduled?

26. A small combination lock has 3 wheels, each labeled with the 10 digits from 0 to 9. If an opening combination is a particular sequence of 3 digits with no repeats, what is the probability of a person guessing the right combination?
27. A software development department consists of 6 women and 4 men.

A) (2pts) How many ways can the department select a chief programmer, a backup programmer, and a programming librarian?

B) (3pts) How many of the selections in part A) consist entirely of women?

C) (3pts) How many ways can the department select a team of 3 programmers to work on a particular project?

28. A card is drawn at random from a standard 52-card deck. Events G and H are:
   G = the drawn card is black;
   H = the drawn card is divisible by 3. (face cards are not valued)

A) Find P(H|G).

B) Test H and G for independence.