Math 315 Midterm Exam 3 Review Material
Closed Book Exam. No calculators allowed.


1. Terminology - Understand and know how to use: power series, radius of convergence, ordinary point, singular point, recurrence relation, system of first order differential equations, homogeneous system, nonhomogeneous system, linear system, general solution, linear independence, fundamental set, Wronskian, phase plane, direction field, phase plane portrait, equilibrium point.

2. Techniques - Be able to use with simple problems:
   - series solution of $n^{th}$ order initial value problems;
   - conversion of $n^{th}$ order initial value problem to first order system;
   - determination of matrix-vector form for linear systems;
   - determination of conditions for existence and uniqueness of solutions for linear systems;
   - checking linear independence for solutions to homogeneous linear systems;
   - solution of constant coefficient linear first order homogeneous systems;
   - determination and classification of equilibrium points;
   - phase plane graph or direction field sketch for constant coefficient linear first order homogeneous systems with $n = 2$. 

- 11.2.5-21
  - 11.2.5: solve \((1 - x)y' + y = 0\) with series.
  - 11.2.7: solve \((x - 4)y' + y = 0\) with series.
  - 11.2.9: solve \((2 - x)y' + 2y = 0\) with series.
  - 11.2.11: solve \((2 - x)y' + 2y = 0\) with series.
  - 11.2.13: solve \(y'' + y = 0\) with series.
  - 11.2.15: solve \(y'' + x^2y = 0\) with series; ordinary point \(x = 0\)?
  - 11.2.17: solve \(y'' + 2xy' - y = 0\) with series; ordinary point \(x = 0\)?
  - 11.2.19: solve \(y'' + xy' - y = 0\) with series; ordinary point \(x = 0\)?
  - 11.2.21: solve \((1 + x)y'' + y = 0\) with series; ordinary point \(x = 0\)?
• 8.1.3-25; 8.4.3-15; 8.5.3-15,19-23 typical problems.
  - 8.1.3: \( x' = -ax + ay; y' = rx - y - xz; \)
    \( z' = -bz + xy; \) autonomous system?
  - 8.1.7: \( x(t) = 2e^{2t} - 2e^{-t}; x(t) = -e^{-t} + 2e^{2t}; \)
    solutions to \( x' = -4x + 6y; y' = -3x + 5y? \)
  - 8.1.13: matrix-vector system for
    \( x'' + dx' - x + x^3 = g \cos(wt)? \)
  - 8.1.21: \( u' = v + \cos(u); v' = v - tw; \)
    \( w' = 5v - 9v + 8w; \) autonomous? vector form?
  - 8.4.5: \( x_1' = x_2; x_2' = x_3; x_3' = -x_1 - x_2 + \sin(t); \)
    homogeneous? linear?
  - 8.4.15: \( t^2 x_1' = -2x_1x_2 + 3x_2; \)
    \( x_2'/t = tx_1 - 4x_2/t; \) homogeneous? linear?
    existence, uniqueness?
  - 8.5.3: \( x_1' = x_1 + x_2; x_2' = -x_1 + x_2; \)
    matrix-vector form?
  - 8.5.5: \( x_1' = x_1 + x_2; x_2' = -x_1 + x_2 + e^t; \)
    matrix-vector form?
  - 8.5.9: \( \mathbf{x}(t) = (e^t \cos(t), -e^t \sin(t))^T; \)
    \( \mathbf{y}(t) = (e^t \sin(t), e^t \cos(t))^T; \) solution to 8.5.3?
  - 8.5.11: given \( \mathbf{x}_p(t) = (e^t, 0)^T; \) and 8.5.9 \( \mathbf{x}, \mathbf{y}, \)
    is \( \mathbf{x}_p + \mathbf{x} + \mathbf{y} \) a solution to 8.5.5?
  - 8.5.19: \( \mathbf{y}_1(t) = (-e^{-t}, -e^{-t}, e^{-t})^T, \)
    \( \mathbf{y}_2(t) = (0, -e^t, 2e^t)^T, \)
    \( \mathbf{y}_3(t) = (e^{2t}, 0, 2e^{2t})^T; \)
    linearly independent? If \( \mathbf{y}(0) = (1, 1, 1)^T \)
    find \( \mathbf{y}(t) = C_1\mathbf{y}_1(t) + C_2\mathbf{y}_2(t) + C_3\mathbf{y}_3(t). \)
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  - 9.1.21: $A = \begin{pmatrix} 7 & 10 \\ -5 & -8 \end{pmatrix}$; find fundamental solution set.
  - 9.1.23: $A = \begin{pmatrix} 5 & -4 \\ 8 & -7 \end{pmatrix}$; find fundamental solution set.
  - 9.2.3: $A = \begin{pmatrix} -2 & 2 \\ 8 & -7 \end{pmatrix}$; find general solution to $y' = Ay$.
  - 9.2.9: if $y(0) = (0, -1)^T$, find solution for 9.2.3.
  - 9.2.17: $A = \begin{pmatrix} -1 & -2 \\ 4 & 3 \end{pmatrix}$; find real general solution.
  - 9.2.23: if $y(0) = (0, 1)^T$, find solution for 9.2.17.
  - 9.2.31: $A = \begin{pmatrix} 3 & -1 \\ 1 & 1 \end{pmatrix}$; find real general solution.
  - 9.2.37: if $y(0) = (2, -1)^T$, find solution for 9.2.31.