11.5 ① one: t. ② three: f(t), g(t), h(t). ③ because its outputs are vectors.

④ subtract p₀ from p₁: \( \langle x₁-x₀, y₁-y₀, z₁-z₀ \rangle \).

⑤ \( \vec{r}(t) = \langle x(t) + (x₁-x₀)t, y(t) + (y₁-y₀)t, z(t) + (z₁-z₀)t \rangle \) ⑥ f component is 0, so \( y = 0 \).

⑦ \( \lim_{t \to a} \vec{r}(t) = \left( \lim_{t \to a} x(t), \lim_{t \to a} y(t), \lim_{t \to a} z(t) \right) \)

⑧ \( \vec{r}(t) \) is continuous at \( t = a \) if and only if \( f, g, h \) are all continuous at \( t = a \).

⑨ \( \begin{align*} x &= 0 + 4t, \quad y = 0 + 7t, \quad z = 1 + 0t, \text{ or } \vec{r}(t) = \langle 4t, 7t, 1 \rangle & \text{(15)} \end{align*} \)

⑩ \( \langle 8, -5, 6 \rangle \to \langle -3, 5, 6 \rangle \) is one possible answer.

⑪ line \( \vec{r}_1 \) has parallel vector \( \langle -2, 8, -4 \rangle \)

line \( \vec{r}_2 \) has parallel vector \( \langle -2, 1, 17 \rangle \) then line will have parallel vector \( \langle -2, 8, -4 \rangle \times \langle -2, 1, 17 \rangle = \langle -4, 6, 14 \rangle \).

⑫ \( y \)-component is constant at 1, so curve lies in plane \( y = 1 \) :

\[ x = 1 - 4t, \quad z = 2 + 6t, \quad t = 3 + 14t \] is my answer.

⑬ Similar to ⑪, for \( \vec{r} \) and \( \vec{r} \) to intersect, we'd need a solution to

\[
\begin{align*}
4 &= -3 - 7s, \quad t = 0, 0, 0 \\
1 - t &= 1 + 9s, \quad 0 = 5, 0, 0 \\
1 + t &= -9 + 4s, \quad 3 = -5, 3, 0 \\
2 + t &= 14 + 7s, \quad 0 = -5, 0, 3
\end{align*}
\]

So there is no solution to \( \vec{r}(t) = \vec{r}(s) \), and \( \vec{r}, \vec{r} \) are not parallel, so they're skew.

⑭ a) This is a line. ① lies in plane \( y = 2 \), parabolic

b) x & y make circle, z fixed at 2. ⑥ x & y make circle, y grows.

c) x & y make circle, z fixed at 2. ⑥ x & y make circle, y grows.

d) y & z make circle, x grows with t.
11.6 ① \( \vec{v}' = \langle f', g', h' \rangle \), ② If \( \vec{v} \) is position, then \( \vec{v}' \) is velocity: gives direction and rate of change of \( \vec{v} \). ③ Unit tan. = \( \frac{\vec{v}'}{|\vec{v}'|} \). ④ \( \vec{v}'' = \langle 90t^8, 0, -\cos t \rangle \).

⑤ \( \int \vec{v}(t) \, dt = \langle \int f(t) \, dt, \int g(t) \, dt, \int h(t) \, dt \rangle \).

⑥ \( \int_a^b \vec{v}(t) \, dt = \langle \int_a^b f(t) \, dt, \int_a^b g(t) \, dt, \int_a^b h(t) \, dt \rangle \).

⑦ \( \vec{v}'(t) = \frac{d}{dt} \langle 2t^3, 6t^2, \frac{3}{t} \rangle = \langle 6t^2, 3t^{-\frac{1}{2}}, -3t^{-2} \rangle = \langle 6t^2, \frac{3}{t}, -\frac{3}{t^2} \rangle \).

⑧ \( \vec{v}' = \frac{d}{dt} \langle 4t, 3\cos 2t, 2\sin 3t \rangle = \langle 0, -6\sin(2t), 6\cos(3t) \rangle \).

⑨ \( \vec{v}' = \frac{d}{dt} \langle e^{-t}, t^2 + t, t \cos t \rangle = \langle -(t+1)e^{-t}, 1 + \ln(t), \cos(t) - t\sin(t) \rangle \).

⑩ \( \vec{v}' = \langle 1, -2\sin 2t, 2\cos t \rangle \); \( \vec{v}'(\pi) = \langle 1, 0, 0 \rangle \).

⑪ \( \vec{v}' = 8t^3 \vec{i} + 9t^2 \vec{j} - \frac{10}{t^2} \vec{k} \); \( \vec{v}'(a) = \langle 8a^3 + 9a^2, -\frac{10}{a^2} \rangle \).

⑫ \( \vec{v}' = 2e^t \vec{i} - 2e^{-2t} \vec{j} + 8e^t \vec{k} \); \( \vec{v}'(0) = \langle 2, -\frac{1}{2}, 7 \rangle \).

⑬ \( \vec{v}' = \langle 1, 0, -3t^2 \rangle \); \( \vec{v}(t) = 1 + \frac{\sqrt{1 + \frac{4}{t^2}}}{t^2 + 4} \).

⑭ \( \vec{y}' = \langle \cos t, -\sin t, -e^t \rangle \); \( \vec{y}'(0) = \langle 1, 0, -1 \rangle \).

⑮ \( \vec{v}(t) = \int \vec{v}'(t) \, dt = \int \langle f(t), g(t), h(t) \rangle \, dt = \langle \frac{3}{2}t^2, \frac{3}{2}\sin t, H(t) \rangle = \langle 3, 0, -2 \rangle \).

⑯ \( \vec{v}(t = 2, 3, 4) \Rightarrow \vec{F} = \langle 2 + C, 2 + D, 0 + E \rangle \), so \( C = \frac{2}{3}, D = 3, E = 4 \) and

(\( \vec{F}(t) = \langle \frac{3}{2}t^2 + \frac{4}{3}, \frac{1}{2}\sin(2t^2), \frac{3}{2} + 4t^2 \rangle \).

11.7 ① vel. = \( \vec{v}' \), speed = \( |\vec{v}'| \), accel. = \( \vec{v}'' \). ② on a circle, \( \vec{F} \perp \vec{v}' \).

③ \( \vec{F} = m\vec{a} \), or \( \vec{F} = m\vec{v}'' \). ④ \( \vec{F} = m \langle 0, 0, -9.8 \rangle \).

⑤ vel. = \( \int \vec{v}(t) \, dt \), use \( \vec{v}(0) \) to determine values of the constants.

⑥ pos. = \( \int \vec{v}(t) \, dt \), use \( \vec{v}(0) \) to determine constants.

⑦ vel. = \( \vec{v}' = \langle -2t, 6t^2 \rangle \), speed = \( |\vec{v}'| = \sqrt{4t^2 + 36t^4} = 2t\sqrt{1 + 9t^2} \), accel. = \( \vec{v}'' = \langle -2, 12t \rangle \).

⑧ vel. = \( \vec{v}' = \langle -26\sin 2t, 24 \cos 2t, 10 \cos 2t \rangle \), speed = \( |\vec{v}'| = \sqrt{26^2 \sin^2 2t + 24^2 \cos^2 2t + 10^2 \cos^2 2t} = \sqrt{676 \sin^2 2t + 676 \cos^2 2t} = \sqrt{676} = \sqrt{26} \), accel. = \( \vec{v}'' = \langle -42 \cos 2t - 98 \sin 2t, -20 \sin 2t \rangle \).

⑨ vel. = \( \int \vec{v}(t) \, dt = \langle \frac{t^2}{2} + C, -e^{-t} + D, t + E \rangle \). \( \vec{v}(0) = \langle 0, 0, 1 \rangle \Rightarrow C = 0, D = 1, E = 1 \). So vel. = \( \langle \frac{t^2}{2} - 1 - e^{-t}, t + E \rangle \). pos. = \( \int \vec{v}(t) \, dt = \langle \frac{t^2}{2} + e^{-t}, e^{-t} + D, \frac{1}{2}t^2 + t + E \rangle \). \( \vec{v}(0) = \langle 0, 0, 1 \rangle \Rightarrow C = 0, D = -1, E = 0 \). So \( \vec{v}(t) = \langle \frac{t^2}{2} + 4, t + e^{-t}, \frac{1}{2}t^2 + t \rangle \).
11.8  ① \[ L = \int_{a}^{b} \sqrt{1 + (\frac{dy}{dx})^2} \, dx = \sqrt{b^2 - a^2} \]  ② \[ L = \int_{a}^{b} \sqrt{(f'(t))^2 + (y'(t))^2 + (z'(t))^2} \, dt \]

③ (Length from \( t = a \) to \( t = b \)) \[ = \int_{a}^{b} \sqrt{\left(\frac{dr}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2} \, dt \]

④ \( \mathbf{v}' = \langle 6t, 8t^2 \rangle \)  \[ \mathbf{v}' = \sqrt{36t^2 + 64t^2} = \sqrt{100t^2} = 10t \text{ for } t > 0 \]
\[ \text{So } L = \int_{0}^{10} 10 \, dt = 100 \]

⑤ \( \mathbf{v}' = \langle x, (2t+1)^2 \rangle \)  \[ \mathbf{v}' = \sqrt{t^2 + (2t+1)^2} = \sqrt{(t+1)^2} = t+1 \text{ for } t+1 > 0 \]
\[ \text{So } L = \int_{0}^{1} (t+1) \, dt = \left[\frac{t^2}{2} + t\right]_0^1 = 2 \]

⑥ \( \mathbf{v}' = \langle 6t^3 - 3t^2, 15t^2 \rangle \)  \[ \text{speed} = \mathbf{v}' = \sqrt{36t^4 + 9t^4 + 225t^4} = \sqrt{270t^4} = \sqrt{3\sqrt{30}} \]
\[ L = \int_{0}^{\frac{\sqrt{270}}{\sqrt{30}}} \sqrt{3\sqrt{30}} \, t^2 \, dt = \sqrt{3\sqrt{30}} \cdot \left[ \frac{\sqrt{30}}{3} \right] = \left[ 26t \right] \text{ for } t > 0 \]
\[ L = \int_{0}^{26} 26 \, dt = 134 \int_{0}^{26} = 134 \]

12.1  ① a point on the plane and a vector normal to the plane.  ② \((-2, -3, 4)\)

③ intersection with xy plane: set \( z = 0 \), get \(-2x - 3y = 12\)

④ \((x-1) + 1(y-0) + 1(z-0) = 0, \text{ or } x+y+z = 1\).

⑤ respectively, they are parallel to the \( x-, \text{ and } y- \) axes.

⑥ traces are curves you see when you cut the surface with planes \( x = k, y = k, z = k \).

⑦ \((x-1) - 1(y-0) + 2(z+3) = 0, \text{ so } x-y+2z = -5\)

⑧ \((4, 5, -1) \quad \langle 5, 0, -2 \rangle \quad \langle 5, 2, 3 \rangle \quad \langle 5, 0, -2 \rangle = \left\langle \frac{5}{2}, -\frac{3}{2}, -2 \right\rangle = \left\langle \frac{5}{2}, 2, 0 \right\rangle \]

⑨ \((2, -1, 1) \quad \langle 6, 2, 3 \rangle \quad \langle 5, 9, -2 \rangle \]

So plane is \( 4(x-1) + 27(y-1) + 10(z+1) = 0, \text{ or } 4x + 27y + 10z = 21 \)
25. Normal vectors are \((1,1,4)\) and \((-1,1,3)\), and \((1,1,4) \cdot (-1,3,1) = 0\), so these planes are perpendicular (orthogonal).

26. Normal vectors are \((2,3,1)\) and \((-10,-8,15)\), and \((-10,-8,15) = -5(2,3,-3)\), so the vectors and the planes are parallel.

31. \(-1(x-1) + 2(y-0) - 4(z-4) = 0\), or \(-x + 2y - 4z + 17 = 0\).

32. \(a: -x + 2y + z = 1\) A direction vector for the line is \((-1,2,1) \times (1,1,1) = (1,2,3)\).

33. \(b: x + y + z = 0\) A point on the line is \((-\frac{1}{2},0,\frac{1}{2})\). So my equation is \(\vec{r}(t) = (-\frac{1}{2} + t, 2t, \frac{1}{2} - 3t)\).

39. \(z = y^2\) is parallel to \(x\)-axis. \(42. x = z^2 - 4\) is parallel to \(y\)-axis.

42. \(x^2 + \frac{y^2}{4} + \frac{z^2}{9} = 1\). Intercepts are \((\pm 1,0,0)\), \((0,\pm 2,0)\), \((0,0,\pm 3)\).

\(xy\)-plane trace is \(x^2 + \frac{y^2}{4} = 1\): \(x\)-plane trace is \(x^2 + \frac{z^2}{9} = 1\):

\(yz\)-plane trace is \(\frac{y^2}{4} + \frac{z^2}{9} = 1\): \(y\)-plane trace is \(\frac{z^2}{9} = 1\):

45. \(z = \frac{x^2}{4} + \frac{y^2}{9}\)

Setting any two of the variables to zero implies that the third must be zero. So only intercept is \((0,0,0)\).

\(xy\)-trace: \(\frac{x^2}{4} + \frac{y^2}{9} = 0 \Rightarrow (0,0)\)

\(xz\)-trace: \(z = \frac{x^2}{4}\)

\(yz\)-trace: \(z = \frac{y^2}{9}\)
\( \frac{y^2}{4} + \frac{z^2}{9} - \frac{x^2}{16} = 1 \). 

\( y = 2 = 0 \) gives no solution for \( x \).
\( x = z = 0 \) gives \( y = \pm 2 \), \( x = y = 0 \) gives \( z = \pm 3 \).

So intercepts are \((0, \pm 2, 0)\) and \((0, 0, \pm 3)\) (no \( x \)-intercepts).

\[ \begin{align*}
\text{xy-trace: } & \frac{y^2}{4} - \frac{x^2}{16} = 1 \\
\text{xz-trace: } & \frac{z^2}{9} - \frac{x^2}{16} = 1 \\
\text{yz-trace: } & \frac{y^2}{4} + \frac{z^2}{9} = 1
\end{align*} \]

63 \( x^2 + \frac{y^2}{4} = z^2 \). Only intercept is \((0, 0, 0)\).

\[ \begin{align*}
\text{xy-trace: } & x^2 + \frac{y^2}{4} = 0 \Rightarrow (0, 0) \\
\text{xz-trace: } & x^2 = z^2 \Rightarrow z = \pm x \\
\text{yz-trace: } & \frac{y^2}{4} = z^2 \Rightarrow z = \pm \frac{y}{2}
\end{align*} \]

Note that the traces \( z = k \) are all ellipses.

63 \( -x^2 + \frac{y^2}{4} - \frac{z^2}{9} = 1 \). Intercepts: no \( z \), no \( x \), \((0, \pm 2, 0)\).

\[ \begin{align*}
\text{xy-trace: } & -x^2 + \frac{y^2}{4} = 1 \\
\text{xz-trace: } & -x^2 - \frac{z^2}{9} = 1, \text{ no solutions.} \\
\text{yz-trace: } & \frac{y^2}{4} - \frac{z^2}{9} = 1
\end{align*} \]
a) $y = z^2$ is parabola projected parallel to $x$-axis, so $\boxed{D}$.  

b) plane, so $\boxed{A}$.  

c) all traces are ellipses, so ellipsoid, so $\boxed{E}$.  

d) traces are ellipses and hyperbolas, so $\boxed{F}$.  

e) $z = k$ traces are ellipses, $z = 0$ is a single point, so $\boxed{B}$.  

f) $y = 1 \times 1$ projected parallel to $z$-axis, so $\boxed{C}$.

12.2  
(1) Dependent (output) is $z$, independent (inputs) are $x, y$.  
(2) all of $\mathbb{R}^2$.  
(3) $xy \neq 0$, so all points in $\mathbb{R}^2$ which do not lie on the $x$- or $y$-axes.  
(4) $x - y \geq 0$, or $x \geq y$:  
(5) three: $x$-, $y$- and $z$-axes.  
(6) Set $z$ equal to various constants and graph the resulting $xy$ relations.  
(7) when $z = 0$, level curve is a single point, $(0, 0)$.  
when $z < 0$, no level curve at all.  
when $z > 0$, ellipses (bigger $z \Rightarrow$ bigger ellipse).  
(8) level surfaces are of the form $f(x, y, z) = k$, so three variables.  
(9) 6 inputs, so $\mathbb{R}^6$.  
(10) $x^2 - y^2$ always defined, $\cos(x^2 - y^2)$ always defined, so domain is $\boxed{\mathbb{R}^2}$.  
(11) $25 - x^2 - y^2 \geq 0 \Rightarrow x^2 + y^2 \leq 25$  
(12) $y^2 - x^2 \neq 0$, so $y \neq \pm x$;  
(13) all points not on $y = x$ or $y = -x$.  
(14) $x^2 - y > 0$, so $y < x^2$.

29  
a) outputs always between 1 and 1, oscillating, so $\boxed{A}$.  
b) trace $x = 0$ is $z = 2 |y|$  
level curves are ellipses  

c) outputs blow up as $x \to y$, level curves are lines $x - y = k$, so $\boxed{B}$.  
d) $p(x, y) = 1$, $p(x, y) \to 0$ as $|x, y| \to \infty$, level curves are circles, so $\boxed{C}$.  

\( x^2 + y^2 = 0 \) : \\
\( x^2 + y^2 = 1 \) : \\
\( x^2 + y^2 = 4 \) :

\[ z = 0 : \ y = x^2 + 1 \]
\[ z = \frac{1}{2} : \ y = x^2 + \frac{5}{4} \]
\[ z = 1 : \ y = x^2 + 2 \]
\[ z = \frac{3}{2} : \ y = x^2 + \frac{13}{4} \]
\[ z = 2 : \ y = x^2 + 5 \]

Imagine cutting horizontally through the surface at various levels:
(a) \( \rightarrow \) B
(b) \( \rightarrow \) E
(c) \( \rightarrow \) C
(d) \( \rightarrow \) D
(e) \( \rightarrow \) A
(f) \( \rightarrow \) F

12.3 \( \text{If } (x_0, y_0) \rightarrow (a, b) \) along any path, \( f(x_0, y_0) \rightarrow L. \)
(3) It means \( \lim_{(x_0, y_0) \rightarrow (a, b)} \text{poly} = \text{poly's output at } (a, b). \)
(5) If outputs approach two different levels along two different paths, then the limit cannot exist.
(6) Because to prove a limit exists, you must get \( f(x_0, y_0) \rightarrow L \) along the infinitely many possible paths.
\[ \lim_{(x,y) \to (6,2)} \left( \frac{x^2 - 3xy^2}{x - 3y} \right) = \lim_{(x,y) \to (6,2)} \left( \frac{x - 3y}{x - 3y} \right) = \lim_{(x,y) \to (6,2)} (x) = 6. \]

28. As \((x,y) \to (6,0)\) along \(x\)-axis, \(f(x,y) = \frac{4x(x)}{3x^2 + (10)^2} = \frac{0}{3x^2} \to 0\) → \(0\) → \(\text{so limit DNE.}\)

As \((x,y) \to (6,0)\) along \(y=x\), \(f(x,y) = \frac{4x(x)}{3x^2 + x^2} = \frac{4x^2}{4x^2} \to 1\) → \(\text{so limit DNE.}\)

29. As \((x,y) \to (0,0)\) along \(x\)-axis, \(f(x,y) = \frac{-2x^2}{x^2} \to -2\)

As \((x,y) \to (0,0)\) along \(y\)-axis, \(f(x,y) = \frac{y^4}{y} \to 0\)

30. Continuous on its domain, which is \(\mathbb{R}^2 - \{(0,0)\}\).

38. Continuous on its domain, which is \(\{(x,y) \mid x \neq 0 \text{ and } y \neq \pm 1\}\)

39. Since \(\frac{xy}{x^2 + y^2}\) is continuous on its domain, which is \(\mathbb{R}^2 - \{(0,0)\}\), \(f\) is continuous at all points other than \((0,0)\).

If \(f\) could be continuous at \((0,0)\), if \(\lim_{(x,y) \to (0,0)} f(x,y)\) exists and is 0.

However, \(\lim_{(x,y) \to (0,0)} \frac{xy}{x^2 + y^2} = \lim_{x \to 0} \frac{0}{x^2} = 0\)

and \(\lim_{(x,y) \to (0,0)} \frac{xy}{x^2 + y^2} = \lim_{x \to 0} \frac{x^2}{x^2 + x^2} = 1\).

So \(\lim_{(x,y) \to (0,0)} f(x,y)\) DNE, and \(f\) is not cont. at \((0,0)\).

So \(f\) is continuous on \(\mathbb{R}^2 - \{(0,0)\}\).
continuous on its domain, which is $\mathbb{R}^2$.

continuous on its domain, which is $x-y>0$, or $x>y$.

continuous on its domain, which is $\mathbb{R}^2$.

continuous on its domain, which is $4-x^2-y^2 \geq 0$

$\Rightarrow x^2+y^2 \leq 4$.

continuous on its domain, which is $\mathbb{R}^2$.

12.4 1) $f_x(a,b)$ is the slope the surface has in the positive $x$ direction.

$f_y(a,b)$ is the slope the surface has in the positive $y$ direction.

2) $f_x = 6xy+y^3$, $f_y = 3x^2+3xy^2$.

3) $f_y = -x^2 \cos(xy) + \cos(xy)$.

4) $f_{xx} = 6y$, $f_{xy} = 6x+2y^2$, $f_{yy} = 6y$, $f_{yx} = 6x+2y^2$.

5) Take deriv. of $f(x,y,z)$ treating $z$ as variable and $x,y$ as constants.

11) $f(x,y) = 3x^2+4y^3$, $f_x = 6x$, $f_y = 12y^2$.

17) $g(x,y) = \cos(2xy)$. $g_x = -2 \sin(2xy)(2y) = -2y \sin(2xy)$, $g_y = -\sin(2xy)(2x)$

19) $f(x,y) = e^{xy}$, $f_x = e^{xy} \cdot \frac{\partial}{\partial x} (xy) = e^{xy}(2xy)$, $f_y = e^{xy} \cdot \frac{\partial}{\partial y} (xy) = e^{xy}(2x)$

21) $f(\omega z) = \frac{\omega}{\omega^2+z^2}$, $f_\omega = \frac{(2 \omega^2 + z^2) \times 1 - \omega(2z)}{(\omega^2+z^2)^2} = \frac{\omega^2 - \omega z}{(\omega^2+z^2)^2}$

$f_z = \frac{\partial}{\partial z} \left( \omega (\omega^2+z^2)^{-1} \right) = \omega (-1)(\omega^2+z^2)^{-2}(2z) = \frac{-2\omega z}{(\omega^2+z^2)^2}$

23) $g(x,z) = x \ln(2z^3x^2)$, $g_z = x \cdot \frac{1}{2z^3+x^2}(2z) = \frac{2xz}{2z^3+x^2}$

$g_x = x \cdot \frac{1}{2z^3+x^2}(2x) + \ln(2z^3x^2).1 = \frac{2x^2}{2z^3+x^2} + \ln(2z^3x^2)$
(23) \( s(y,z) = z^2 \tan(yz) \). \[
\frac{ds}{dy} = z^2 \sec^2(yz) + 2z \tan(yz),
\]
\[
\frac{ds}{dz} = 2z \sec^2(yz) y + \tan(yz),
\]
\[2z = \frac{2z^3 \sec^2(yz) + 2z \tan(yz)}{yz^3}.
\]

(24) \( F(p,q) = (p^2 + pq + q^2)^{\frac{1}{2}} \).
\[
F_p = \frac{1}{2} (p^2 + pq + q^2)^{-\frac{1}{2}} \cdot (2p + q)
\]
\[
= \frac{2p + q}{2 \sqrt{p^2 + pq + q^2}}.
\]

Similarly, \( F_q = \frac{p + 2q}{2 \sqrt{p^2 + pq + q^2}} \).

(4) \( g(x,y,z) = 2x^2y - 3x^2z + 10y^2z^2 \).
\[
g_x = 4xy - 3x^2, \quad g_y = 2x^2 + 20yz, \quad g_z = -12x^2z + 20y^2z.
\]

(5) \( h(w,x,y,z) = \frac{w^2}{xy} \).
\[
\frac{dh}{dw} = \frac{1}{x} \cdot \frac{w^2}{xy} \cdot \frac{1}{w} = \frac{w}{xy} \cdot \frac{1}{w} = \frac{w}{xy},
\]
\[
\frac{dh}{dx} = \frac{1}{x} \cdot \frac{w^2}{y} \cdot x^{-1} = \frac{w^2}{y} \cdot (-x^{-2}) = \frac{-w^2}{x^2y},
\]
\[
\frac{dh}{dy} = \frac{1}{y} \cdot \frac{w^2}{x^2} \cdot y^{-1} = \frac{-w^2}{xy^2}.
\]

(6) \( f(x,y) = 1 - \arctan(x^2 + y^2) \).
\[
f_x = -\frac{1}{1 + (x^2 + y^2)^2} \cdot \frac{2x}{x^2 + y^2} = \frac{-2x}{1 + (x^2 + y^2)^2}.
\]
\[
f_y = -\frac{1}{1 + (x^2 + y^2)^2} \cdot \frac{2y}{x^2 + y^2} = \frac{-2y}{1 + (x^2 + y^2)^2}.
\]